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CONTENTS

A. J. KLUYVER and J. C. HOOGERHEIDE: "On the influence of mono iodo acetic acid on the respiration and the fermentation of yeasts", p. 596.

A. J. KLUYVER and J. C. HOOGERHEIDE: "On the presumed suitability of maltose as a respiration substrate for non-maltose fermenting yeasts", p. 605.

P. ZEEMAN and J. DE GIER: "Preliminary note on some experiments concerning isotopes of some of the noble gases and hydrogen by means of J. J. THOMSON's mass spec rograph", p. 609. (With one plate).

W. H. KEESOM and Miss A. P. KEESOM: "Isopycnals of liquid helium". II, p. 612.

A. EINSTEIN und W. MAYER: "Spaltung der natuerlichsten Feldgleichungen fuer Semi-Vektoren in Spinor-Gleichungen vom DIRAC'schen Typus", p. 615.

J. M. BURGERS: "On the application of statistical mechanics to the theory of turbulent fluid motion", VII, p. 620.

F. A. H. SCHREINEMAKERS and H. H. SCHREINEMACHERS: "Osmosis in systems consisting of water and tartaric acid and containing three liquids, separated by two membranes", II, p. 629.

F. M. JAEGER and J. E. ZANSTRA: "The Allotropism of Beryllium", p. 636.

A. H. BLAAUW: „Temperatuur en Tijd van den Bloemaanleg bij Bol-Irissen", p. 644.

G. VAN RIJNBERK and J. TEN CATE: "Musculo-muscular "story" (étage") reflexes of dogs whose spinal chord has been cut in several places", p. 653.

J. M. TIENSTRA: "Nets of Triangles Consisting of Points with Circular Error-Curves". (Communicated by Prof. F. A. VENING MEINESZ), p. 656.

A. G. VAN VEEN and W. K. MERTENS: "On the isolation of a toxic bacterial pigment". (Provisional communication). (Communicated by Prof. P. VAN ROMBURGH), p. 666.

E. H. BUCHNER and H. E. STEUTEL: "The osmotic pressure and the viscosity of nitro-cellulose solutions". (Communicated by Prof. A. F. HOLLEMAN), p. 671.

B. G. ESCHER: "On the relation between the volcanic activity in the Netherlands East Indies and the belt of negative gravity anomalies discovered by VENING MEINESZ". (Communicated by Prof. F. A. VENING MEINESZ), p. 677.

R. W. VAN BEMMELEN: "Die Anwendung der Undationstheorie auf das alpine System in Europa". (Communicated by Prof. G. A. F. MOLENGRAAFF), p. 686.

A. RUBINO: "The Arterial Supply of the Cerebral Cortex in a Chinese Brain". (Communicated by Prof. C. U. ARIËNS KAPPERS), p. 694. (With three plates).

JOSEPH L. SHELLSHEAR: "The Arterial Supply of the Cerebral Cortex". (Communicated by Prof. C. U. ARIËNS KAPPERS), p. 700. (With one plate).
M. P. DE BRUYN OUBOTER: "Mengentäuschungen im taktil-kinaestethischen Gebiet". (Communicated by Prof. B. BROUWER), p. 710.

Microbiology. — *On the influence of mono iodo acetic acid on the respiration and the fermentation of yeast.* By A. J. KLUYVER and J. C. HOOGERHEIDE.

(Communicated at the meeting of June 24, 1933).

§ 1. *Introduction.*

According to the insight of many investigators, amongst whom PFLÜGER and PFEFFER may be considered to be the pioneers, the respiration process, in which sugars act as a respiration substrate, is characterized bij an initial anaerobic phase and the oxygen only reacts with products of the anaerobic breakdown of the sugar. This initial phase would then be identical with the initial phase occurring in the completely anaerobic breakdown of the sugar, i.e. in the fermentation process.

Contrary to this unitarian theory many investigators hold that from the chemical point of view respiration and fermentation are quite independent processes. Although they are willing to accept a physiological relation between the two, they reject the idea of a common phase in both.

In recent years this view has gained many fresh supporters chiefly by the important observations of LUNDSGAARD regarding the influence of mono iodo acetic acid on the living cell [1]). The experiments of LUNDSGAARD which have been fully corroborated by many other authors leave no doubt that the addition of the said acid in a suitable concentration to the medium of the cells may fully inhibit the fermentation whilst the respiration is maintained at its normal intensity.

At first sight this result seems to be absolutely incompatible with the view that the first phase of respiration is identical with that of fermentation, since in this case the suppression of the anaerobic breakdown of the sugar would also prevent the respiration.

It is especially BOYSEN JENSEN who of late has emphasized this aspect of LUNDSGAARD's work [2]).

To appreciate fully the weight of this argument it is perhaps not superfluous to remark that the reverse, i. e. the specific prevention of respiration with the preservation of fermentation, had already be attained in several ways a.o. by addition of *HCN*, *CO* and *NO* to the medium of the

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[1]) E. LUNDSGAARD, Biochem. Zeitschr. **217**, 162, 1930; Ibid. **220**, 1 und 8, 1930; Ibid. **227**, 51, 1930; Ibid. **250**, 61, 1932.

[2]) P. BOYSEN JENSEN, Biochem. Zeitschr. **236**, 211, 1931.

cells. This fact, however, could easily be explained by assuming that the oxygen which in respiration reacts with the products of the initial anaerobic breakdown of the substrate asks for a special activating apparatus. The destruction or temporary inactivation of the latter would then lead to the mentioned effect.

The same line of argument cannot be very well applied for the explanation of the action of the mono iodo acetic acid, since one would expect that the inhibition of the fermentation will involve also the suppression of the first phase of respiration.

Since at the other hand so many facts point in favour of the unitarian theory of respiration and fermentation, it was the aim of the experiments reported in this communication to make a closer study of the action of mono iodo acetic acid in the hope to find data which allowed to reconcile the observations made with the said theory.

§ 2. *Present views regarding the influence of mono iodo acetic acid on respiration and fermentation.*

LUNDSGAARD (l. c.) himself has concluded that his experiments have brought decisive evidence that respiration and fermentation are fully independent from the chemical point of view. In proof of this contention the following summary of one of his papers may be cited: [1])

„In der Monojodessigsäurevergiftung verfügen wir über eine Methode, die uns die Trennung des Spaltungs- und des Oxydations-Stoffwechsels ermöglicht, in dem die Monojodessigsäure den ersteren aufzuheben vermag ohne den letzteren nennenswert zu beeinflussen."

Already in his first paper LUNDSGAARD gives as his opinion that the inhibition of the anoxybiotic breakdown occurs in one of the first steps of the reaction chain. In this connection he points to the fact that in fermentation experiments with poisoned yeast no disappearance of the sugar could be observed. Moreover he states that with poisoned zymase preparations no phosphorylation takes place. YAMASAKI [2]) also concludes that the effect of mono iodo acetic acid on the fermentation must be ascribed to its destructive action on the phosphorylating agent.

BARRENSCHEEN and BRAUN [3]) derive from the results of their experiments that it is chiefly the more or less complete destruction of the coenzyme which is responsible for the disappearance of the glycolytic activity of muscle under the influence of the mono iodo acetic acid. This view is rejected by LOHMANN [4]) since the latter could prove that the weakening effect of mono iodo acetic acid on the lactic acid formation by muscle

[1]) E. LUNDSGAARD, Biochem. Zeitschr. **220**, 18, 1930.
[2]) I. YAMASAKI, Biochem. Zeitschr. **228**, 123, 1930.
[3]) H. K. BARRENSCHEEN und K. BRAUN, Biochem. Zeitschr. **232**, 165, 1931.
[4]) K. LOHMANN, Biochem. Zeitschr. **236**, 444, 1931.

extract was also manifest when hexose biphosphoric ester or methylglyoxal were used as substrates.

However the effect of the poison in the case of the methylglyoxal conversion was much less pronounced than that on the glycolytic activity. For this reason LOHMANN also critisizes the view of DUDLEY [1]) who concluded from his experiments that the checking of the glycolysis would be due to the inactivation of the methylglyoxalase. LOHMANN himself thinks it probable that the action of the mono iodo acetic acid is caused by a general weakening of the glycolytic ferment system as a whole.

At the other hand NILSSON, ZEILE and VON EULER [2]) are inclined to accept the view that the action of mono iodo acetic acid is restricted to a final phase of fermentation not essential to respiration. Their chief argument is rather of an indirect nature and is based on the fact that a clear cut separation of respiration and fermentation in yeast can only be brought about by applying a special concentration of the halogenic acid. They show that somewhat higher concentrations of the acid also affect respiration, which proves that the difference between the two dissimilation processes is not as fundamental as it seems at first sight.

BOYSEN JENSEN [3]) then proved that no accumulation of any product of anaerobic sugar breakdown in experiments, in which yeast cells had been poisoned by a suitable concentration of mono iodo acetic acid, could be detected. This result may be considered incompatible with the theory of NILSSON and co-workers.

In a study on the autoxidation of thiol compounds BERSIN [4]) made the interesting suggestion that the poisoning of glycolysis in muscle would be due to the removal of the reduced component of the glutathione system by the mono iodo acetic acid.

In a recent paper QUASTEL and WHEATLEY [5]) have given special attention to this point. These authors have indeed shown that mono iodo acetic acid reacts with the physiological thiol compounds cysteine and reduced glutathione. In agreement herewith the addition of the said compounds diminished the toxic action of mono iodo acetic acid on brain tissue. The importance of this observation is now considerably increased by the demonstration that cysteine and glutathione cause a very marked increase in the rate of aerobic fermentation by yeast cells, whilst glutathione has practically no influence on the respiration. QUASTEL concludes that thiol compounds, normally present in the yeast cells, play some controling part in the relationship between respiration and fermentation.

This conclusion is materially supported by the earlier observations of

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[1]) H. W. DUDLEY, Biochem. Journ. **25**, 439, 1931.

[2]) R. NILSSON, K. ZEILE und H. VON EULER, Zeitschr. f. physiol. Chemie **194**, 53, 1931.

[3]) P. BOYSEN JENSEN, Biochem. Zeitschr. **236**, 211. 1931.

[4]) TH. BERSIN, Biochem. Zeitschr. **248**, 3, 1932.

[5]) J. H. QUASTEL and A. H. M. WHEATLEY, Biochem. Journ. **26**, 2169, 1932.

BUMM and APPEL [1]) who clearly showed that glutathione, whilst having no effect on respiration and anaerobic glycolysis by tumour cells, markedly increases the aerobic glycolysis.

With these facts in mind it indeed does not seem excluded that the destruction of reduced glutathione, normally occurring in the living cell, by mono iodo acetic acid plays an important part in the specific toxic action of this acid on fermentation.

§ 3. *Hypothesis regarding the nature of the action of mono iodo acetic acid based on the unitarian theory of respiration and fermentation.*

As was stated in the introduction many facts point in favour of the view that respiration and fermentation are characterized by the identity of the first steps of their respective reaction chains. Elsewhere one of us (Kl.) has expressed this situation in this way, that fermentation may be considered as a process in which the function of the oxygen in respiration has been taken over by other hydrogen acceptors and inversely that respiration is a fermentation process which has been led in other channels by the intervention of oxygen after activation of this element by WARBURG's respiration ferment [2]).

This view is materially supported by the results of an analysis of the mechanism of fermentation and respiration, since the most probable representation of this mechanism is that both processes consist of primary reactions which are all of the oxidoreduction type. This result is especially important since it makes acceptable that all these primary reactions occur under the influence of one and the same catalyst of the nature of an oxidoreductase [3]).

Is there now any possibility to reconcile the behaviour of mono iodo acetic acid with regard to the yeast cell with this unitarian theory?

The facts reported in the foregoing paragraph seem to make this rather difficult. For it is not easy to understand that a poison can suppress fermentation completely without influencing respiration in the least. At first sight one would expect that both processes would be injured approximately to the same degree.

However the definition of respiration as a fermentation process led into new channels by the intervention of the activated oxygen seems to open a possibility for another explanation. For we may conceive that the rate of respiration is determined by the velocity with which oxygen is made available by the ironcontaining activating apparatus. If we then add the supposition that the activated oxygen is always preferent as a hydrogen acceptor in the dehydrogenation of methylglyoxal, it is clear that a decrease

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[1]) E. BUMM und H. APPEL, Zeitschr. f. physiol. Chemie, **210**, 79, 1932.

[2]) A. J. KLUYVER, Archiv f. Mikrobiologie **1**, 181, 1930.

[3]) A. J. KLUYVER, Chemie der Zelle und Gewebe **13**, 134, 1926.

of the rate of anaerobic breakdown of the sugar will only affect respiration at the moment, that the reduction is so large that the demand of the oxygen can no longer be satisfied. From this point of view fermentation under aerobic conditions is only the consequence of the excess of sugar breakdown as compared with the need for dehydrogenation substrate of the respiration process.

We may in this line of thought assume that the action of increasing concentrations of mono iodo acetic acid present in the medium of the yeast cells will lead to an inactivation of an increasing part of the oxidoreduction catalyst. At first this inactivation will only affect the surplus dissimilation, i.e. the aerobic fermentation. A gradual increase of the concentration of the poison will lead at last to a complete suppression of the aerobic fermentation, the respiration maintaining its normal intensity. However as soon as this critical concentration has been surpassed the capacity of the still remaining sugar breakdown will fall below the level that is necessary for the maintenance of the normal respiration rate. In other words respiration too will become affected.

Although in the paper of NILSSON, ZEILE und VON EULER indications are found that the experimental facts are in agreement with this mode of view, it did not seem to be superfluous to reinvestigate this point in more detail, the more so because the Swedish investigators draw quite different conclusions from their experiments.

§ 4. *Experiments on the influence of various concentrations of mono iodo acetic acid on respiration and fermentation.*

As outlined above it was the aim of the experiments to study the effect of the mono iodo acetic acid in concentrations in the neighbourhood of the critical concentration which just suppresses aerobic fermentation. For this purpose it was essential to study the various factors which influence the toxic action of the acid.

In all these experiments the wellknown manometric method of WARBURG has been applied. For the measurement of the respiration and the aerobic fermentation always two parallel experiments were made in such a way, that in one of these experiments caustic potash was brought in the inner tube of the vessel for the absorption of the carbon dioxide formed. At the same time a third experiment was made in which the air was replaced by an atmosphere of oxygenfree nitrogen to which 5 % of carbon dioxide had been added, in order to saturate the medium with this gas. From this experiment the rate of anaerobic fermentation could be derived.

In all cases a suspension was made of about 400 mgrs baker's yeast in 100 ccm of a 2.5 % phosphate buffer solution to which also 1 gram of glucose and the mono iodo acetic acid in varying quantities were added. A measured quantity of this mixture was brought in the WARBURG appa-

ratus and this was shaken in a waterbath of 30° C for about 20 minutes before readings were made.

The experiment itself lasted for one hour and from the observed changes in pressure the values of respiration (Q_{O_2}), of aerobic fermentation ($Q^{O_2}_{CO_2}$) and of anaerobic fermentation ($Q^{N_2}_{CO_2}$) were calculated in the usual way.

In all cases baker's yeast ("Koningsgist" kindly supplied by the "Nederlandsche Gist- en Spiritusfabriek") has been used. All comparative experiments were made with one and the same sample of yeast, which if necessary was kept in a refrigerator. Special experiments showed that under these conditions the intensity of respiration and fermentation remained practically constant during several days.

Preliminary experiments showed, in agreement with the results of LUNDSGAARD and of EHRENFEST [1]), that the toxic action of the mono iodo acetic acid markedly increased with increasing acidity of the medium. It was then decided to carry out the experiments at a $pH = 4.7$, since a separate set of experiments had shown that this acidity was practically optimal for both fermentation and respiration of the variety of yeast employed.

The influence of mono iodo acetic acid in several concentrations varying between $\frac{1}{100.000}$ and $\frac{1}{7000}$ was then studied.

The results of these experiments are given in Table I.

TABLE I.

The effect of increasing concentrations of mono iodo acetic acid on respiration, aerobic and anaerobic fermentation of *Saccharomyces cerevisiae* in a medium with $p_H = 4.7$.

Concentration of CH_2ICOOH	Q_{O_2}	$Q^{N_2}_{CO_2}$	$Q^{O_2}_{CO_2}$
0	123.2	279.7	88.1
1 : 100.000	127.2	121.1	68.6
1 : 65.000	120.1	81.6	...
1 : 50.000	122.4	63.9	30.5
1 : 40.000	127.0	0.9	11.8
1 : 30.000	104.9	—	1.6
1 : 20.000	70.8	—	—
1 : 15.000	43.7	—	—
1 : 10.000	27.3	—	—
1 : 7.000	12.0	—	—

[1]) E. EHRENFEST, Journ. of Biol. Chem. **97**, 1932; Proc. Am. Soc. of Biol. Chem. 26th Ann. Meeting p. LXXVI.

In Fig. 1 a graphical representation of these results is given. In this graph the quantities of sugar consumed in respiration, aerobic and anaerobic fermentation are given. These quantities were calculated from

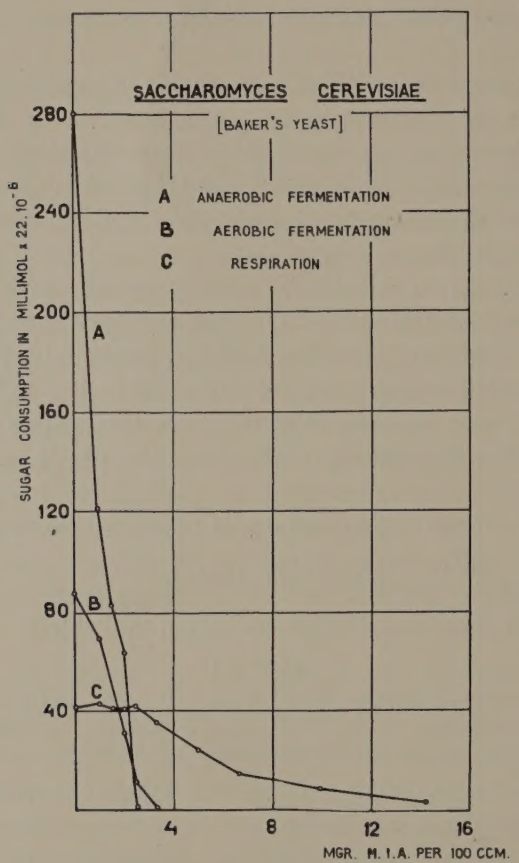

Fig. 1.

the figures for oxygen consumption and carbon dioxide evolution, on the basis of 1 molecule of sugar consuming 6 molecules of O_2 in respiration and one molecule of sugar yielding 2 molecules of carbon dioxide in fermentation.

The figures on the ordinate are of an arbitrary character (100 representing 0.0022 millimol sugar).

When we look upon the graph, we find in the first place a confirmation of LUNDSGAARD's fundamental observation, viz. that on raising the concentration of the mono iodo acetic acid anaerobic fermentation is rather quickly inhibited, whilst at first respiration is maintained at its original level. At a concentration of 2.5 mgr. per 100 ccm (1 : 40.000) anaerobic fermentation is altogether checked, the respiration still remaining constant. As for the aerobic fermentation we see that this process too is quickly inhibited by

increasing concentrations of the poison and at the critical concentration already mentioned its value has practically been reduced to zero. However the remarkable fact presents itself that as soon as this critical concentration of the poison has been surpassed also the intensity of respiration is affected. It is obvious that this result is in excellent agreement with the „surplus-theory" of aerobic fermentation. As soon as the surplus breakdown of the sugar has practically disappeared and therefore aerobic fermentation has dropped out, a further increase in concentration of the poison should also lower the rate of respiration.

This situation throws a special sidelight on LUNDSGAARD's pronouncement that mono iodo acetic acid offers a specific means of bringing about a clear cut separation between oxybiotic and anoxybiotic metabolism. For it is clear that this behaviour of the yeast cells can be very well explained on the basis of the unitarian theory, viz. by assuming an inactivation of increasing parts of the oxidoreductase by increasing concentrations of the acid, the respiration always being preferent for the conversion of the remaining part of the supply of anaerobic breakdown products.

Against this mode of view the objection can be raised that at the critical concentration of the acid the undiminished intensity of respiration proves that still a fraction of the oxidoreductase is active, so that one would expect that under these conditions also a fraction of the anaerobic fermentation would be maintained. It is however tempting to connect this apparent deviation with the observations of QUASTEL and WHEATLEY that at the one hand mono iodo acetic acid reacts with the reduced form of glutathione and that at the other hand the latter compound is a more or less essential condition for the normal course of the fermentation process. QUASTEL is inclined to ascribe the function of the glutathione to its property to establish a suitable oxidation reduction potential in the cell and we should like to point out in addition that the earlier experiments of BOYLAND [1]) give rather conclusive proof that the establishment of such a suitable reduction potential is an essential condition for the fermentation process.

This might well be the cause that at the critical concentration of the acid under anaerobic conditions the still remaining part of the oxidoreductase does not make itself manifest by evolution of fermentation carbon dioxide.

It seemed possible to give a still more convincing demonstration of the correctness of the „surplus-theory" of aerobic fermentation by using other yeast species in which the intensity of this fermentation was lower. If a yeast species could be found in which under aerobic conditions the fermentation would be completely suppressed, one should expect that any addition of the mono iodo acetic acid large enough to affect the anaerobic fermentation would also diminish at once the rate of respiration. It is obvious that such a demonstration would greatly derogate LUNDSGAARD's postulate of the independence of oxybiotic and anoxybiotic metabolism.

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[1]) E. BOYLAND, Biochem. Journ. **24**, 703, 1930.

It may be stated immediately that until now we did not succeed in finding a yeast species which under the aerobic conditions, as realized in

TABLE II.

The effect of increasing concentrations of mono iodo acetic acid on respiration, aerobic and anaerobic fermentation of *Saccharomyces Marxianus* in a medium with $p_H = 4.7$.

Concentration of CH_2ICOOH	Q_{O_2}	$Q^{N_2}_{CO_2}$	$Q^{O_2}_{CO_2}$
0	92.3	122.5	9.7
1 : 100.000	74.2	61.5	...
1 : 50.000	68.5	21.7	4.4
1 : 30.000	53.7	10.7	2.9
1 : 20.000	48.4	—	—
1 : 10.000	13.8	—	—

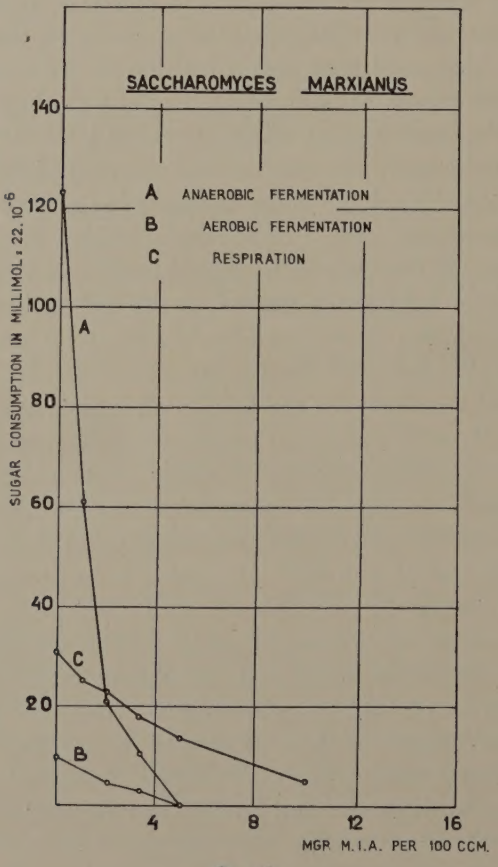

Fig. 2.

the WARBURG apparatus, was fully devoid of aerobic fermentation, whilst showing a good anaerobic fermentation. However in the course of this investigation we met with some species which approximately answered the said requirements viz. in which the aerobic fermentation was very low.

We will give here only the results of the investigation of one of these yeast species: *Saccharomyces Marxianus* Hansen.

The experiments were carried out in exactly the same way as has been described before for the baker's yeast with the understanding that in these cases three days old cultures on yeast extract glucose plates were used.

Table II gives the figures for oxygen consumption and carbon dioxide production by *Saccharomyces Marxianus* expressed in the usual way. (See table II, pag. 604.)

The results are also graphically reproduced in Figure 2 in the same way as has been done before in Figure 1.

As may be derived from Fig. 2 we meet in *Saccharomyces Marxianus* with a yeast with a very low aerobic fermentation. Even very small concentrations of the mono iodo acetic acid will practically reduce the „surplus breakdown" to nihil and therefore it should be expected that in this case respiration will be almost immediately affected. As will be seen from the graph the experimental results are in perfect agreement with this assumption.

It seems to us that the foregoing exposition justifies the conclusion that LUNDSGAARD's discovery of the action of mono iodo acetic acid on yeast cells does not suffice to reject the unitarian theory of respiration and fermentation.

Microbiology.—*On the presumed suitability of maltose as a respiration substrate for non-maltose fermenting yeasts.* By A. J. KLUYVER and J. C. HOOGERHEIDE.

(Communicated at the meeting of June 24, 1933).

In a paper published about two years ago by TRAUTWEIN and WEIGAND [1]) a new argument against the unitarian theory of respiration and fermentation was raised. In this paper the authors report about experiments which tend to show that for some yeast species which lacked the ability to ferment the disaccharide maltose this sugar still was a suitable substrate for the respiration process.

It is clear that this demonstration would imply that the chemistry of the maltose respiration proceeds by a direct oxidation of this sugar, since a preliminary hydrolysis of the disaccharide to glucose and a subsequent

[1]) K. TRAUTWEIN und K. WEIGAND, Biochem. Zeitschr. **240**, 423, 1931.

anaerobic breakdown to the C_3-stage would be incompatible with the lack of ability of the said yeast species to ferment maltose. The evidence given might be considered as a serious objection against the idea that respiration and fermentation have a common introductory phase. It seemed therefore worthwile to investigate whether the results of TRAUTWEIN and WEIGAND could be corroborated.

A closer study of their paper showed that there was some reason to doubt the correctness of the interpretation of their observations. In the first place our attention was drawn to the fact that the German authors had applied highly concentrated solutions of the respiration substrates. As a matter of fact they had worked with 10 % solutions of the various sugars. Now it had already been shown by MEYERHOF [1]) for a *Torula* species that the rate of respiration is highly independent of the concentration of the respiration substrate. MEYERHOF found for instance (c.f. Tabelle VIII) that the respiration value for this yeast was the same in a 0.5 % and in a 5 % glucose medium. We learn from this result that already low concentrations of suitable substrates raise the rate of respiration to its optimal value.

If it could be shown that this effect was of quite general occurrence it might be deemed to be of far reaching importance, also for the problem under consideration. For this would mean that special attention should be given to impurities eventually present in substrates in those cases in which these were applied in high concentrations.

Our first aim was therefore to make a study of the influence of sugar concentration on the rate of respiration.

The technique used was again the manometric method of WARBURG. For particulars regarding the arrangement of the experiments we refer to our preceding paper [2]). Ordinary baker's yeast was used.

The influence of varying the glucose concentration from 10 % to 0.05 % was studied. The results are given in Table 1.

We learn from these results that indeed the rate of respiration is highly independent of the glucose concentration, in so far as only a reduction of the concentration of the sugar below 0.2 % gives a rate which is markedly reduced as compared with that in 10 % sugar. It even should be remarked, that a slight optimum in the rate of respiration is present in the region between 0.5—0.2 % glucose.

With a view to the above, it is obvious that it is not recommendable to study the suitability of some respiration substrate in high concentrations since then the danger, that positive results are due to some impurity possibly present, is markedly increased.

It was decided therefore to study the respiration of some non-maltose fermenting yeasts in media with varying concentrations of glucose and

[1]) O. MEYERHOF, Biochem. Zeitschr. **162**, 43, 1925.

[2]) These Proceedings **36**, p. 596, 1933.

TABLE 1.

The effect of various concentrations of glucose on the respiration of Saccharomyces cerevisiae (baker's yeast) in a medium with $p_H = 4.7$.

% glucose	Q_{O_2}
10	107.5
5	109.5
3	108.9
1	107.5
0.5	118.7
0.35	128.5
0.20	115.0
0.10	80.0
0.05	33.7
—	15.0

maltose respectivily. For these experiments use was made of pure cultures of *Saccharomyces Marxianus* HANSEN and of *Saccharomyces exiguus* REESS, these being the same species as employed by TRAUTWEIN and WEIGAND.

The results are given in Table 2.

TABLE 2.

The effect of various concentrations of glucose and maltose on the respiration of Saccharomyces Marxianus and of Saccharomyces exiguus.

	Saccharomyces Marxianus	Saccharomyces exiguus
	Q_{O_2}	Q_{O_2}
Without sugar	29.1	27.0
Glucose 1%	91.9	76.1
Glucose 10%	89.9	67.4
Maltose 1%	50.7	36.2
Maltose 10%	89.4	90.2

These figures give a confirmation of the results of TRAUTWEIN and WEIGAND in so far that indeed the respiration in a 10 % solution of maltose is equal to or even higher than that in a 10 % solution of glucose.

Moreover we learn from these figures that here also the rate of respiration is practically independent of the concentration of the glucose. In contrast herewith, however, the rate of respiration in maltose is markedly decreased by lowering the concentration of this sugar.

This latter result may be considered to be a strong indication that indeed the high rate of respiration in the maltose medium is due to some impurity present in the sugar preparation employed. If this assumption was correct it might be expected that on continuing the respiration experiments for a longer period this impurity would be consumed with a corresponding decrease of the respiration rate.

To test this theory the experiments reported in Table 3 were made.

TABLE 3.

The respiration of Saccharomyces Marxianus and of Saccharomyces exiguus in 10% glucose and in 10% maltose solutions for four successive hours.

Period	Saccharomyces Marxianus		Saccharomyces exiguus	
	Q_{O_2}		Q_{O_2}	
	10% glucose	10% maltose	10% glucose	10% maltose
1st hour	89.9	89.4	67.4	90.2
2nd „	93.1	54.6	70.7	83.0
3rd „	85.1	24.0	67.7	63.5
4th „	91.4	20.6	68.1	27.3

It is obvious that the results reported in Table 3 give an excellent confirmation of the view that the respiration in maltose containing media is due to some impurity. Whilst the rate of respiration in the glucose media remains practically constant during the whole course of the experiment, the respiration in the maltose media shows a continuous decline. In the last hour of the experiment the respiration has fallen down to the same level as in the experiments without sugar (c.f. Table 2). Since the sugar consumption in experiments like these is negligible we may conclude that the yeast cells do not show any increase in respiration even in the presence of high concentrations of maltose.

In all the foregoing experiments a commercial brand of maltose had been used, i.e. a preparation of Merck, as had been also employed by TRAUTWEIN and WEIGAND. It seemed therefore worthwhile to investigate the behaviour of other maltose preparations.

Although a chemical pure maltose of the firm of Pfanstiehl gave a much lower respiration value than the maltose of Merck and some other commercial brands, there was still a clear increase in respiration with all these products as compared with the respiration in the absence of any sugar.

Since the same was found to apply for the fermentation it became

probable that the impurity which was present in small quantities in the maltose preparations would be a fermentable sugar. It was therefore decided to purify the maltose by adding to a 10 % solution a suspension of a glucose — but not maltose — fermenting yeast. As such *Saccharomyces Marxianus* was used. After four hours incubation at 30° C. the yeast cells were separated from the maltose solution by filtration through a Seitz filter. The sterile filtrate was then used for a new series of respiration experiments.

TABLE 4.

Purified maltose against crude maltose as a substrate for the respiration of Saccharomyces Marxianus and of Saccharomyces exiguus.

	Saccharomyces Marxianus	Saccharomyces exiguus
	Q_{O_2}	Q_{O_2}
Without sugar	29.1	27.0
10 % maltose (Merck)	89.4	90.2
10 % maltose purified	30.8	36.4

As will be seen from Table 4 the rate of respiration of both species in the purified maltose solution is practically identical with that in the absence of sugar.

We may therefore conclude that pure maltose is unsuitable as a substrate for the respiration of *Saccharomyces Marxianus* and *Saccharomyces exiguus*.

Herewith the arguments given by TRAUTWEIN and WEIGAND in favour of the dualistic theory of respiration and fermentation have been refuted.

Physics. — *Preliminary note on some experiments concerning isotopes of some of the noble gases and hydrogen by means of* J. J. THOMSON's *mass spectrograph.* By P. ZEEMAN and J. DE GIER.

(Communicated at the meeting of June 24, 1933).

Sir JOSEPH THOMSON's beautiful parabola method, the original method of positive ray analysis depending on the use of parallel magnetic and electric fields has been much refined by ASTON but has still some advantages of its own.

We have built a mass spectrograph according to the original THOMSON pattern with some improvements according to CONRAD[1]) and some slight modifications of our own necessitated by our purpose in view: the determination of the different kinds of atoms and molecules in the discharge tube with only short expositions.

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[1]) R. CONRAD. Phys. Z. S. **31**, 888. 1930.

A more detailed description must be reserved for a future communication.

Some photographs were made with the inert gases *He*, *Ne*, and *Ar*. Besides the well known parabolas, rather weak ones were observed for $\frac{m}{e} = 5{,}23$ and 41. They are probably due to hydrides of the inert gases, according to a suggestion of Sir J. J. THOMSON [2]) and seconded by ASTON [3]) [4]). We studied these hydrides under various circumstances, and intend to return to the subject.[4a])

Prof. KEESOM [5]) was kind enough to supply us with a sample of a mixture of hydrogen isotopes. The sample contained 3% of $H^1 H^2$ molecules. With our rather wide slits a parabola $\frac{m}{e} = 4$ was obtained after a very short time of exposure. A new parabola $\frac{m}{e} = 5$ was observed after an illumination of about 40 minutes. The $\frac{m}{e} = 4$ corresponds to the $(H_2^1 H^2)^+$ ion, the $\frac{m}{e} = 5$ corresponds to a $(H^1 H_2^2)^+$ ion. A parabola $\frac{m}{e} = 6$ is to be expected; it would correspond to a $(H_3^2)^+$ ion, but the probability for the formation of a triatomic hydrogen ion of heavy atoms only, is so small, that we have not to be astonished that a three hours exposition does not show any trace of the $\frac{m}{e} = 6$ parabola. [6])

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[2]) J. J. THOMSON. Proc. Royal Soc. **99**, 90, 1921.

[3]) F. W. ASTON. Isotopes 1924, pag. 117.

[4]) H. LUKANOW und W. SCHÜTZE. Z. S. f. Phys. **82**, 610. 1933 again pronounce the same idea.

[4a]) Experiments with mixtures of hydrogen on oxygen with the rare gases prove that there is very probably no isotope of neon of mass 23, and no isotope of argon of mass 41. A second criterion to distinguish between isotopes and hydrides of the same mass is given by the phenomenon of multiple charges. The presence of oxygen increases the appearance of multiply charged ions of the rare gases. Even with potentials of only 16—20 K V, doubly ionised *He*, quadruply ionised *Ne*, and quadruply on quintriply ionised *Ar*, are easily observed. Rare gas hydrides never exhibit these multiply charged states. The hydrogen isotopes give a means of studying the hydrides in another direction.

[5]) W. H. KEESOM, H. VAN DIJK, J. HAANTJES Proc. Amsterdam **36**. 248. 1933, see also the note [1]); in their publication, pag. 252.

[6]) While this paper was in the press Prof. KEESOM kindly supplied us with some 3 cm^3 containing at least 10% $H^1 H^2$ molecules. We obtained now the parabola $\frac{m}{e} = 6$ corresponding to the $(H^2 H^2 H^2)^+$ ion with an exposure of less than one hour.

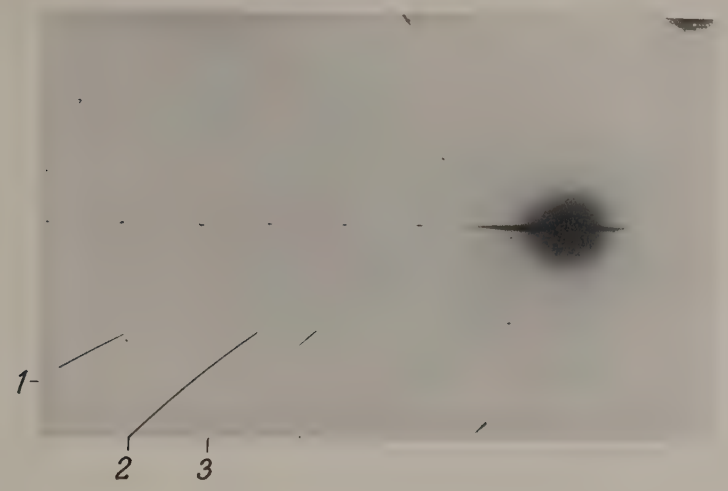

Fig. 1.

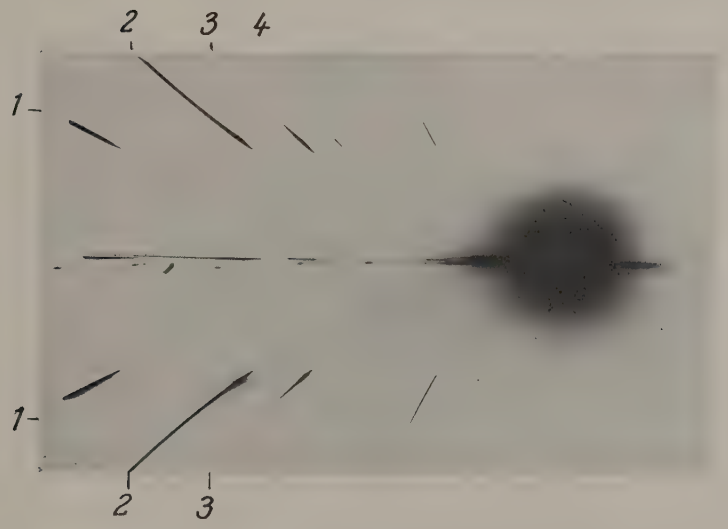

Fig. 2.

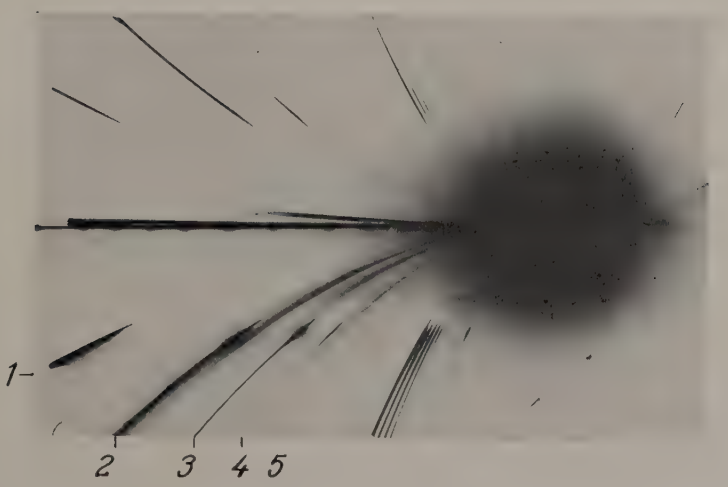

Fig. 3.

The normal triatomic hydrogen ion is produced directly by the process [7])

$$H_2^+ + H_2 \rightarrow H_3^+ + H$$

according to the balance of evidence.

If the molecule or ion contains a H atom we may obtain a triatomic ion with $\frac{m}{e} = 4$ [8])

$$H^1 H^{2+} + H_2^1 \rightarrow H_2^1 H^{2+} + H^1$$

$$H_2^{1+} + H^1 H^2 \rightarrow H_2^1 H^{2+} + H^1$$

For an ion with $\frac{m}{e} = 5$ we find the production given by

$$H^1 H^{2+} + H^1 H^2 \rightarrow H^1 H_2^{2+} + H^1$$

$$H_2^{2+} + H_2^1 \rightarrow H^1 H_2^2 + H^1$$

$$H_2^2 + H_2^{1+} \rightarrow H^1 H_2^2 + H^1$$

Hence it follows that high pressure and great current intensity must be favourable for the production of the different triatomic ions. The pressure in the discharge tube must be rather high, because the available tension is low. The current intensity could not be varied independently of the pressure: more intense current demands higher pressure. It therefore turned out that there was no advantage increasing the current intensity as much as possible. The relative abundance might then increase, but the smaller free path length at the same time increases the probability of destruction of the produced ion. The pressure between the fine slits is also increased by leak from the discharge tube. Hence the geometry of the apparatus determines an optimum for pressure and current intensity. [9]) It appeared to be of importance that the tube was held free from oxygen. The "hydrides" of hydrogen apparently are rather easily oxidized. The best way is to wash with hydrogen till the parabolas of the hydrocarbons of the unavoidable greasy vapours do not exhibit prolongations. (If there is no oxygen present, these only rarely indicate double charges). Further impurities appeared then no more confusing.

DESCRIPTION OF PLATE.

Fig. 1. Short exposition with normal hydrogen.

Fig. 2. At the top, hydrogen with admixture of heavy isotope.
At the bottom, ordinary hydrogen (electric field reversed).

Fig. 3. At the top, hydrogen with heavy isotope admixed.
At the bottom, the same with long exposure. (Voltage 17 K. V., current 3 m.A.). On the original the H_5 line is clearly seen. In the reproduction it is undoubtedly visible as a trace.

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[7]) H. D. SMYTH, Rev. of modern Physics **3**, 347. 1931.

[8]) K. T. BAINBRIDGE, Phys. Rev. **42**, 1. 1932.

[9]) BRASEFIELD, Phys. Rev. **31**, 52. 1928.

Physics. — *Isopycnals of liquid helium.* II. By W. H. KEESOM and Miss A. P. KEESOM. (Comm. N°. 224*e* from the KAMERLINGH ONNES Laboratory at Leiden).

(Communicated at the meeting of June 24, 1933).

Summary. Measurements on the isopycnals of liquid helium were continued to 35 atmospheres. Part of the solidification curve could be determined more accurately. The position of the λ-point in this curve was found to be: $T = 1.753°$ K, $p = 29.91$ atm. Undercooling of liquid helium I was observed.

§ 1. *Introduction.* In a former paper [1]) we communicated a number of measurements on isopycnals of liquid helium up to 25 atm. We now have continued those measurements up to 35 atm., especially in the neighbourhood of the solidification curve and the upper part of the λ-curve.

We tried to make measurements with the same apparatus also in the solid state. This appeared, however, not to be possible, the measurements becoming irregular, probably in consequence of holes being formed in the solid matter. In one case we happened to obtain the metastable production of a liquid helium isopycnal into the solid helium range.

In other cases when cooling the liquid helium (the pressure being sufficiently high) solidification sharply set in at a definite temperature. So we were able to determine part of the solidification curve more accurately than could be done in the measurements dealt with in Comm. No. 184*b* [2]).

For the method and for details we refer to Comm. No. 224*d*.

§ 2. *The results* have been collected in tables I and II and are represented in Fig. 1.

An isopycnal such as No. XII consists of 3 parts, belonging to liquid helium I, to liquid helium II, and to a mixture of liquid helium II and solid helium respectively, the latter part coinciding with the solidification curve. Isopycnal XI is very characteristic, the part belonging to liquid helium II having become very short.The inserted figure on a larger scale shows this part more clearly.

The points marked ⚠ of isopycnal IX correspond to supercooled liquid helium I (cf. § 1).

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[1]) These Proceedings **36**, 482, 1933. Comm. Leiden No. 224*d*.

[2]) These Proceedings **35**, 794, 1926. Comm. Leiden No. 184*b*.

TABLE I.

Isopycnals of liquid helium. Measurements of May 10, 1933.

IX $\varrho = 0.1817$ gr/cm³				X $\varrho = 0.1760$ gr/cm³	
T °K	p atm.	T °K	p atm.	T °K	p atm.
2.519	34.70	1.443	26.12[2])	2.501	26.78
2.306	34.04	1.359	25.78[2])	2.301	26.20
2.082	33.27	1.178	25.29[2])	2.108	25.58
1.890	32.83	1.352	25.62[2])	1.861	25.23
1.843	32.72[1])	1.531	26.71[2])	1.790	25.99
1.780	32.63[1])	1.593	27.25[2])	1.745	26.67
1.747	32.63[1])	1.698	28.70[2])	1.694	27.20
1.708	28.89[2])	1.724	29.24[2])	1.639	27.75
1.635	27.76[2])	1.904	32.83	1.595	27.32[2])
1.590	27.25[2])	2.094	33.32	1.511	26.60[2])
1.543	26.84[2])	2.501	34.66	1.358	25.83[2])
1.477	26.28[2])			1.146	25.27[2])

[1]) Undercooled liquid.
[2]) Solidification curve.

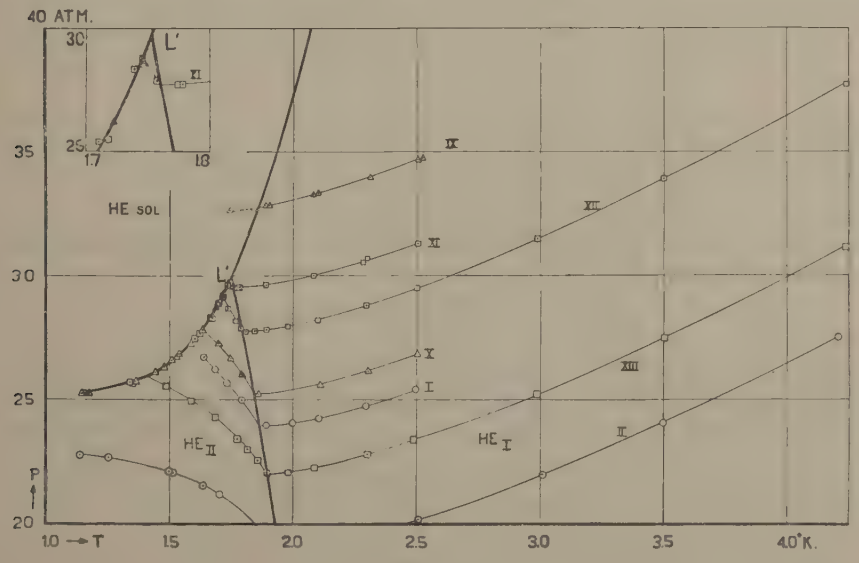

Fig. 1.

TABLE II.

Isopycnals of liquid helium. Measurements of May 18, 1933.

XI $\varrho = 0.1793$ gr/cm³		XII $\varrho = 0.1780$ gr/cm³		XIII $\varrho = 0.1733$ gr/cm³	
T °K	p atm.	T °K	p atm.	T °K	p atm.
2.509	31.33	1.891	27.83	4.233	31.20
2.301	30.73	1.844	27.77	3.504	27.47
2.080	30.02	1.809	27.75	2.987	25.18
1.890	29.60	1.792	27.85	2.498	23.31
1.778	29.55	1.770	28.15	2.297	22.83
1.758	29.58	1.741	28.68	2.087	22.25
1.746	29.76	1.717	29.10	1.984	22.05
1.738	29.67[1])	1.696	28.87[1])	1.891	22.05
1.710	29.08[1])	1.670	28.32[1])	1.858	22.54
1.621	27.64[1])	1.608	28.48[1])	1.819	23.00
1.705	28.92[1])	1.980	27.92	1.778	23.42
1.739	29.67[1])	2.096	28.20	1.689	24.32
1.774	29.55	2.293	28.85	1.595	24.95
1.894	29.59	2.503	29.50	1.486	25.52
2.280	30.55	2.987	31.48	1.344	25.74[1])
2.502	31.35	3.497	33.92	1.157	25.29[1])
		4.233	37.75		

[1]) Solidification curve.

Table III gives data for the solidification curve read from the diagram.

TABLE III.

T °K	p atm.	T °K	p atm.
1.15	25.25	1.50	26.49
1.20	25.34	1.60	27.38
1.30	25.55	1.70	28.87
1.40	25.90	1.75	29.86

A remarkable point is that for densities and temperatures at which the helium was partly solid partly liquid, the solidification curve was accurately followed with rising as well as with falling temperature, for so far as the solid-liquid helium II part of the curve was concerned.

This was, however, not the case for the solid-liquid helium I part of the curve. At falling temperature we decidedly observed an undercooling of the liquid [1]), as already mentioned, whereas at rising temperature equilibrium very slowly was established and sometimes no definite point was reached.

Using isopycnal XI we could fix the point L' where the λ-curve meets the melting curve, *i. e.* the λ-point in the solidification curve. Its position is:

$$T = 1.753^\circ \text{ K}, \quad p = 29.91 \text{ atm.}$$

Even at 1.18° K isopycnal IX, for which the density was highest, does not leave the solidification curve to enter the solid region. We conclude that at 1.18° K the density of solid helium exceeds 0.1817 gr/cm^3. As from the diagram of isopycnals we derive that the density of liquid helium II is 0.1725 gr/cm^3, we conclude that the difference in density between solid helium and liquid helium II at 1.18° K surpasses 0.0092 gr/cm^3. From this follows that the melting heat at that temperature surpasses 0.016 cal/gr. How much these values are surpassed, cannot be derived from these measurements.

[1]) Undercooling or the reverse when passing the λ-curve was never observed.

CORRIGENDUM.

Proc. Academy Amsterdam Vol. **36** p. 486 Fig. 2: in stead of Kg/cm^2 read atm.

Physics. — *Spaltung der natuerlichsten Feldgleichungen fuer Semi-Vektoren in Spinor-Gleichungen vom* DIRAC*'schen Typus.* Von A. EINSTEIN und W. MAYER.

(Communicated at the meeting of June 24, 1933).

In einer frueheren Mitteilung [1]) haben wir gezeigt, dass die allgemeinsten Semivektor-Gleichungen einfachster Art sich in eine kanonische Form bringen lassen, in welcher nur drei willkuerliche Konstante auftreten.

Es zeigte sich ferner, dass die DE BROGLIE-Welle eines solchen Systems sich in zwei Wellentypen von Spinor-Charakter aufspalten laesst, welche sich zwanglos als dem Elektron bezw. Proton zugehoerig deuten lassen.

[1]) Akademie der Wissenschaften, Amsterdam 1933.

Es hat sich nun weiter gezeigt, dass diese Spaltung nicht auf die De Broglie-Wellen beschraenkt ist, sondern dass sie einer ganz allgemeinen Eigenschaft des Gleichungs-Systems entspricht. Dieses letztere zerfaellt naemlich in die zwei Spinor Systeme (16), (17), in welchen die elektrischen Glieder entsprechend (13), (13′) zugefuegt zu denken sind. Die einzige Verbindung zwischen den Systemen (16) und (17) liegt in der Gemeinsamkeit des elektrischen Potentialvektors φ_τ; die Spinorfelder der Elektronen und die der Protonen sind demnach abgesehen von ihrer elektrodynamischen Wechselwirkung voellig unabhaengig voneinander. Entsprechendes gilt gemaess (15) fuer die Stromdichte. Dagegen lassen sich — wenn man an einer Feldtheorie festhalten will — die De Broglie-Felder der einzelnen Partikeln gleicher Art nicht in analoger Weise voneinander trennen.

Die Diracgleichungen in ihrer kanonischen Gestalt

$$\left.\begin{aligned} E^\tau{}_{\sigma\tau}(\psi^\sigma{}_{,\tau} - i\varepsilon\psi^\sigma\varphi_\tau) &= \overline{C}_{\tau\rho}\chi^\rho \\ E^\tau{}_{\sigma\tau}(\chi^\tau{}_{,\tau} - i\varepsilon\chi^\tau\varphi_\tau) &= C_{\rho\sigma}\psi^\rho \end{aligned}\right\} \quad \ldots\ldots\ (1)$$

wo

$$\dot{E} = E(1,0,0,0), \quad C_{\rho\sigma} = i a g_{\rho\sigma} + b v_{\rho\sigma}. \quad \ldots\ldots\ (1')$$

ist, mit

$$v_{\dot{\rho}\dot{\sigma}} \equiv \left\| \begin{matrix} 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ i & 0 & 0 & 0 \end{matrix} \right\| \quad \ldots\ldots\ (2)$$

lassen sich in einfachster Weise in zwei Systeme von Spinorcharakter spalten.

Durch den obigen numerisch invarianten Semitensor $v_{\dot{\rho}\dot{\sigma}}$ erster Art kann man einem jeden Semivektor erster Art $\eta^{\dot{\nu}}$ den Sternvektor $\eta^{*\dot{\nu}}$ vermittels

$$\eta^*_{\dot{\mu}} = v_{\dot{\mu}\dot{\nu}}\eta^{\dot{\nu}} \quad \ldots\ldots\ (3)$$

zuordnen. Man beweist aus $v_{\tau\sigma}v^\sigma{}_\rho = -g_{\tau\rho}$ leicht, dass

$$(\eta^*_{\dot{\mu}})^* = -\eta^*_{\dot{\mu}} \quad \ldots\ldots\ (3')$$

ist. Es gibt nun Semi-Vektoren, deren zugeordnete Sternvektoren ihnen bis auf einen Faktor gleich sind. Dieser Faktor kann nach (3′) nur $\pm i$ sein. Es gibt dementsprechend auch zwei Arten solcher Vektoren: der α-Spinor ($\underset{\alpha}{\varrho}_{\dot{\mu}}$ bezeichnet) definiert durch $\underset{\alpha}{\varrho}_{\dot{\mu}}{}^* = i\,\underset{\alpha}{\varrho}_{\dot{\mu}}$, resp.

$$\underset{\alpha}{\varrho}_{\dot{\mu}} = -i v_{\dot{\mu}\dot{\nu}}\underset{\alpha}{\varrho}_{\dot{\nu}} \quad \ldots\ldots\ (4)$$

und der β-Spinor, definiert durch $\tau^{-}_{\dot{\beta}^{\mu}}{}^{*} = -i\,\tau^{-}_{\dot{\beta}^{\mu}}$, resp.

$$\tau^{-}_{\dot{\beta}^{\mu}} = i\,v^{--}_{\mu\nu}\,\tau^{-}_{\dot{\beta}^{\nu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4')$$

Ausfuehrlich lauten die Relationen

$$\varrho^{-}_{\alpha^{\dot{1}}} = \varrho^{-}_{\alpha^{\dot{4}}},\ \varrho^{-}_{\alpha^{\dot{3}}} = i\,\varrho^{-}_{\alpha^{\dot{2}}};\ \tau^{-}_{\dot{\beta}^{\dot{1}}} = -\tau^{-}_{\dot{\beta}^{\dot{4}}},\ \tau^{-}_{\dot{\beta}^{\dot{3}}} = -i\,\tau^{-}_{\dot{\beta}^{\dot{2}}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Ganz analog definiert man die α resp. β-Spinoren zweiter Art $\bar{\varrho}^{=}_{\alpha^{\mu}}$, $\bar{\tau}^{=}_{\dot{\beta}^{\mu}}$ durch die (4), (4'), resp. (5) konjungiert komplexen Relationen:

$$\bar{\varrho}^{=}_{\alpha^{\dot{1}}} = \bar{\varrho}^{=}_{\alpha^{\dot{4}}},\ \bar{\varrho}^{=}_{\alpha^{\dot{3}}} = -i\,\bar{\varrho}^{=}_{\alpha^{\dot{2}}};\ \bar{\tau}^{=}_{\dot{\beta}^{\dot{1}}} = -\bar{\tau}^{=}_{\dot{\beta}^{\dot{4}}},\ \bar{\tau}^{=}_{\dot{\beta}^{\dot{3}}} = i\,\bar{\tau}^{=}_{\dot{\beta}^{\dot{2}}} \quad . \quad . \quad . \quad (5')$$

Der konjugiert komplexe Semi-Vektor eines α resp. β-Spinors erster Art ist somit ein solcher zweiter Art und umgekehrt. Man beweist (durch Sternbildung)

a. dass die Summe eines α und eines β Spinors nur Null ist, wenn der α und der β Spinor verschwindet. Ebenso zeigt man aus (5), (5'), dass

b. das innere Produkt zweier α (resp. β) Spinoren verschwindet.

Weiter gilt dann die (nach dem gesagten eindeutige) Zerlegung des beliebigen Semi-Vektors $\eta^{-}_{\dot{\sigma}}$:

$$2\,\eta^{-}_{\dot{\sigma}} = (\eta^{-}_{\dot{\sigma}} - i\,\eta^{*}_{\dot{\sigma}}) + (\eta^{-}_{\dot{\sigma}} + i\,\eta^{*}_{\dot{\sigma}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in den α Spinor $\eta^{-}_{\dot{\tau}} - i\,\eta^{-}_{\dot{\tau}}{}^{*}$ und den β Spinor $\eta^{-}_{\dot{\tau}} + i\,\eta^{-}_{\dot{\tau}}$ (Der Spinorcharakter wird durch Sternbildung bewiesen).

Ebenso wie der Semi-Vektor ($\eta^{-}_{\dot{\tau}}$ in (6) hat jeder Semi-Tensor (in Bezug auf jeden seiner Semi-Indizes) eine α, β-Spaltung. So gilt auch in verstaendlicher Bezeichnung

$$E^{r}{}_{\sigma\tau} = (E^{r}_{\underset{\alpha\alpha}{\sigma\tau}} + E^{r}_{\underset{\beta\beta}{\sigma\tau}}) + (E^{r}_{\underset{\alpha\beta}{\sigma\tau}} + E^{r}_{\underset{\beta\alpha}{\sigma\tau}}) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Wir zeigen jetzt, dass in der Zerlegung (7) fuer $E = E\,(1, 0, 0, 0)$ der zweite geklammerte Term rechts verschwindet. In der Tat folgt aus der Form des $E\,(1, 0, 0, 0)$ und den definierenden α, β-Relationen (5), (5'), dass fuer beliebige $\underset{\dot{\beta}}{\varrho^{-}_{\dot{\tau}}}$, $\underset{\alpha}{\pi^{=}_{\dot{\tau}}}$ resp. $\underset{\alpha}{\varrho^{-}_{\dot{\sigma}}}$, $\underset{\dot{\beta}}{\pi^{=}_{\dot{\tau}}}$

$$E^{r}_{\underset{\dot{\beta}\ \alpha}{\dot{\sigma}\dot{\tau}}{}^{=}_{=}}\,\varrho^{-}_{\dot{\beta}}\,\pi^{=}_{\alpha} = 0, \quad . \quad . \quad . \quad . \quad (8) \qquad E^{r}_{\underset{\alpha\ \dot{\beta}}{\dot{\sigma}\dot{\tau}}{}^{=}_{=}}\,\varrho^{-}_{\alpha}\,\pi^{=}_{\dot{\beta}} = 0 \quad . \quad . \quad . \quad . \quad (8')$$

ist. (Man beweist (8) sofort fuer $r = 1$, damit aber gilt (8) fuer $r = 1, \ldots 4$).

Multipliziert man (7) mit $\underset{\dot{\beta}}{\varrho^{-}_{\dot{\sigma}}}\,\underset{\alpha}{\pi^{=}_{\dot{\tau}}}$, so erhaelt man demnach (wegen (*b*))

$$E^{r}_{\underset{\alpha\beta}{\dot{\sigma}\dot{\tau}}{}^{=}_{=}}\,\underset{\dot{\beta}}{\varrho^{-}_{\dot{\sigma}}}\,\underset{\alpha}{\pi^{=}_{\dot{\tau}}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Damit aber verschwindet $E^{r}_{\underset{\alpha\beta}{\dot{\sigma}\dot{\tau}}{}^{=}_{=}}\,\varrho^{-}_{\dot{\sigma}}\,\pi^{=}_{\dot{\tau}}$ fuer beliebige Semi-Vektoren

$\varrho^{\bar{\tau}} = \underset{\alpha}{\varrho}^{\bar{\sigma}} + \underset{\beta}{\varrho}^{\bar{\sigma}}$, $\pi^{\bar{\bar{\tau}}} = \underset{\alpha}{\pi}^{\bar{\bar{\tau}}} + \underset{\beta}{\pi}^{\bar{\bar{\tau}}}$; also ist $E^{\tau}_{\underset{\alpha}{\bar{\sigma}}\,\underset{\beta}{\bar{\bar{\tau}}}} = 0$. Genau so zeigt man, dass auch $E^{\tau}_{\underset{\beta}{\bar{\sigma}}\,\underset{\alpha}{\bar{\bar{\tau}}}} = 0$ ist. Statt (7) gilt somit

$$E^{\tau}_{\sigma\tau} = E^{\tau}_{\underset{\alpha\alpha}{\sigma\tau}} + E^{\tau}_{\underset{\beta\beta}{\sigma\tau}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7')$$

Es ist also $E^{\tau}_{\sigma\tau}\,\underset{\alpha}{\psi}^{\sigma}$ in Bezug auf τ ein β-Tensor ($E^{\tau}_{\underset{\alpha\alpha}{\sigma\tau}}\,\underset{\alpha}{\psi}^{\sigma} = 0$ nach (b)) u.s.w. Beachten wir, dass $C_{\varrho\sigma}$, $\bar{C}_{\varrho\sigma}$ wieder den α resp. β Charakter einer Semigroesse nicht aendert, so zerfaellt, sobald wir setzen

$$\psi^{\sigma} = \underset{\alpha}{\psi}^{\sigma} + \underset{\beta}{\psi}^{\sigma}, \qquad \chi^{\tau} = \underset{\alpha}{\chi}^{\tau} + \underset{\beta}{\chi}^{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

das System (1) in die beiden Systeme

$$\left.\begin{aligned} E^{\tau}_{\underset{\alpha}{\sigma\tau}}\,(\underset{\alpha}{\psi}^{\sigma}{}_{,\tau} - i\,\varepsilon\,\underset{\alpha}{\psi}^{\sigma}\,\varphi_{\tau}) &= \bar{C}_{\tau\rho}\,\underset{\beta}{\chi}^{\rho} \\ E^{\tau}_{\underset{\beta}{\sigma\tau}}\,(\underset{\beta}{\chi}^{\tau}{}_{,\tau} - i\,\varepsilon\,\underset{\beta}{\chi}^{\tau}\,\varphi_{\tau}) &= C_{\rho\sigma}\,\underset{\alpha}{\psi}^{\rho}, \end{aligned}\right\} \quad (11) \qquad \left.\begin{aligned} E^{\tau}_{\underset{\beta}{\sigma\tau}}\,(\underset{\beta}{\psi}^{\sigma}{}_{,\tau} - i\,\varepsilon\,\underset{\beta}{\psi}^{\sigma}\,\varphi_{\tau}) &= \bar{C}_{\tau\rho}\,\underset{\alpha}{\chi}^{\rho} \\ E^{\tau}_{\underset{\alpha}{\sigma\tau}}\,(\underset{\alpha}{\chi}^{\tau}{}_{,\tau} - i\,\varepsilon\,\underset{\alpha}{\chi}^{\tau}\,\varphi_{\tau}) &= C_{\rho\sigma}\,\underset{\beta}{\psi}^{\rho} \end{aligned}\right\} \quad (11')$$

Infolge (1') und der definierenden Relationen (4) gilt

$$C_{\rho\sigma}\,\underset{\alpha}{\psi}^{\rho} = i\,(a-b)\,\underset{\alpha}{\psi}_{\sigma}, \qquad C_{\rho\sigma}\,\underset{\beta}{\psi}^{\rho} = i\,(a+b)\,\underset{\beta}{\psi}_{\sigma} \quad . \quad . \quad . \quad (12)$$

und ebenso

$$\bar{C}_{\tau\rho}\,\underset{\alpha}{\chi}^{\rho} = -\,i\,(a+b)\,\underset{\alpha}{\chi}_{\tau}, \qquad \bar{C}_{\tau\rho}\,\underset{\beta}{\chi}^{\rho} = -\,i\,(a-b)\,\underset{\beta}{\chi}_{\tau} \quad . \quad . \quad (12')$$

Wir erhalten damit die endgueltige Gestalt des DIRAC-Systems

$$\left.\begin{aligned} E^{\tau}_{\underset{\alpha}{\sigma\tau}}\,(\underset{\alpha}{\psi}^{\sigma}{}_{,\tau} - i\,\varepsilon\,\underset{\alpha}{\psi}^{\sigma}\,\varphi_{\tau}) &= -\,i\,(a-b)\,\underset{\beta}{\chi}_{\tau} \\ E^{\tau}_{\underset{\beta}{\sigma\tau}}\,(\underset{\beta}{\chi}^{\tau}{}_{,\tau} - i\,\varepsilon\,\underset{\beta}{\chi}^{\tau}\,\varphi_{\tau}) &= i\,(a-b)\,\underset{\beta}{\psi}_{\sigma}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\left.\begin{aligned} E^{\tau}_{\underset{\beta}{\sigma\tau}}\,(\underset{\beta}{\psi}^{\sigma}{}_{,\tau} - i\,\varepsilon\,\underset{\beta}{\psi}^{\sigma}\,\varphi_{\tau}) &= -\,i\,(a+b)\,\underset{\alpha}{\chi}_{\tau} \\ E^{\tau}_{\underset{\alpha}{\sigma\tau}}\,(\underset{\alpha}{\chi}^{\tau}{}_{,\tau} - i\,\varepsilon\,\underset{\alpha}{\chi}^{\tau}\,\varphi_{\tau}) &= i\,(a+b)\,\underset{\beta}{\psi}_{\sigma}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13')$$

Bildet man das konjugiert komplexe Gleichungs-System zu (13'), so wird es mit (13) identisch, wenn man $\overline{\underset{\alpha}{\chi}}^{\sigma}$ durch $\underset{\alpha}{\psi}^{\sigma}$, $\overline{\underset{\beta}{\psi}}^{\sigma}$ durch $\underset{\beta}{\chi}^{\sigma}$, $a+b$ durch $a-b$ und ε durch $-\varepsilon$ ersetzt.

Beide Systeme unterscheiden sich also, wie zu erwarten ist, tatsaechlich

nur durch den Massenwert und das Vorzeichen von ε. Auch der Strom-Vektor

$$I^r = E^r{}_{\sigma\tau}\,\psi^\sigma\,\bar{\psi}^\tau - E^r{}_{\sigma\tau}\,\bar{\chi}^\sigma\,\chi^\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

erhaelt wegen (8) eine voellig analoge α, β-Spaltung

$$\varepsilon\, I^r = \varepsilon\,(E^r{}_{\sigma\tau}\,\underset{\alpha}{\psi}{}^\sigma\,\underset{\alpha}{\bar{\psi}}{}^\tau - E^r{}_{\sigma\tau}\,\underset{\beta}{\bar{\chi}}{}^\sigma\,\underset{\beta}{\chi}{}^\tau) + (-\varepsilon)\,(E^r{}_{\sigma\tau}\,\underset{\alpha}{\bar{\chi}}\,\underset{\alpha}{\chi}{}^\tau - E^r{}_{\sigma\tau}\,\underset{\beta}{\psi}{}^\sigma\,\underset{\beta}{\bar{\psi}}{}^\tau), \quad (15)$$

wobei wegen (13) die Divergenz jedes der beiden Teilstroeme fuer sich verschwindet. Schreibt man bei Unterdrueckung des elektromagnetischen Potentials die Gleichungen (13) ausfuehrlich an, wobei noch jeweils die dritten und vierten Komponenten (nach (5), (5′)) durch die zweiten und ersten ausgedrueckt sind, so erhaelt man das DIRAC-System:

$$\left.\begin{aligned}
(\psi^1{}_{/1} - \psi^1{}_{/4}) + (\psi^2{}_{/2} + i\,\psi^2{}_{/3}) &= -\,i\,(a-b)\,\chi^1 \\
(\psi^1{}_{/2} - i\,\psi^1{}_{/3}) - (\psi^2{}_{/1} + \psi^2{}_{/4}) &= -\,i\,(a-b)\,\chi^2 \\
(\chi^1{}_{/1} + \chi^1{}_{/4}) + (\chi^2{}_{/2} + i\,\chi^2{}_{/3}) &= \;\;i\,(a-b)\,\psi^1 \\
(\chi^1{}_{/2} - i\,\chi^1{}_{/3}) - (\chi^2{}_{/1} - \chi^2{}_{/4}) &= \;\;i\,(a-b)\,\psi^2
\end{aligned}\right\} \text{ für } \psi = \underset{\alpha}{\psi},\ \chi = \underset{\beta}{\chi}. \quad . \quad (16)$$

entsprechend gibt (13′)

$$\left.\begin{aligned}
(\psi^1{}_{/1} + \psi^1{}_{/4}) + (\psi^2{}_{/2} - i\,\psi^2{}_{/3}) &= -\,i\,(a+b)\,\chi^1 \\
(\psi^1{}_{/2} + \psi^1{}_{/3}) - (\psi^2{}_{/1} - \psi^2{}_{/4}) &= -\,i\,(a+b)\,\chi^2 \\
(\chi^1{}_{/1} - \chi^1{}_{/4}) + (\chi^2{}_{/2} - i\,\chi^2{}_{/3}) &= \;\;i\,(a+b)\,\psi^1 \\
(\chi^1{}_{/2} + \chi^1{}_{/3}) - (\chi^2{}_{/1} + \chi^2{}_{/4}) &= \;\;i\,(a+b)\,\psi^2
\end{aligned}\right\} \text{ für } \psi = \underset{\beta}{\psi},\ \chi = \underset{\alpha}{\chi}. \quad . \quad (17)$$

Diese Zerlegung besagt aber nicht, dass sich nun die Semi-Vektoren-Theorie als unnoetig erwiesen haette, da sie in ihrer endgueltigen (einfachsten) Formulierung reine Spinoren-Systeme aufweist. Denn abgesehen von der (in einer frueheren Arbeit gezeigten) Einbau-Moeglichkeit in das Gebaeude der allgemeinen Relativitaet, die eine reine Spinorentheorie so zwanglos nicht kennt, ist vom Standpunkt der Spinorentheorie nicht verständlich, warum es in der Natur gerade *zwei* verschiedene elementare Trägheitsmassen mit abgesehen vom Vorzeichen gleich grosser elektrischer Ladung gibt.

Hydrodynamics. — *On the application of statistical mechanics to the theory of turbulent fluid motion.* VII. [1]) By J. M. BURGERS. (Mededeeling No. 26 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft).

(Communicated at the meeting of June 24, 1933).

11. *Critical remarks.*

In reconsidering the way which has been followed in the attempt to obtain a statistical description of the turbulent motion of a fluid, several points can be noticed which are open to criticism.

I. In the first place it is certain that the restriction to twodimensional motion will have a considerable influence upon the results; in particular numerical results may come out grossly in error. The restriction has been introduced for the purpose of mathematical simplification. It has been mentioned before (see Part IV, footnote [1]), p. 279) that according to modern views vortices with axes parallel to the main direction of flow (which cannot appear in twodimensional motion) probably will play the most important part. The principal difficulty of the three dimensional case is connected with the description of the field of motion, as the stream function cannot be used. Taking again the motion between two parallel walls, and assuming the y-axis perpendicular to the walls, the x-axis in the direction of the mean motion, and the z-axis perpendicular to both of them, it is possible that a way of attacking the three dimensional case may be found by introducing a form of FOURIER development with respect to both x and z for the components of the velocity of the relative motion. It would be desirable that this development in some way or other was adapted to the representation of vortices stretched out parallel to the direction of the mean flow. If one should succeed in finding a suitable form of expansion, then the equations governing the problem could be written down in a similar way as was followed before, and the exponent of the distribution function could be formed. Again a variational problem might be introduced, in order to obtain a development for the functions of the coordinate y occurring as coefficients in the FOURIER expansion. Naturally the differential equations to be derived would be of a more complicated nature, and probably would contain several unknown functions simultaneously.

II. There is something unsatisfactory connected with the calculation of the mean motion $\overline{U}$. It will be remembered that the mean motion

[1]) Part VI has appeared in these Proceedings, **36**, p. 487, 1933.

was derived from a distribution function, depending upon the function λ; hence, once a choice for λ having been made, it appeared that the mean motion was completely determined. The method, however, which was used to test whether the choice made for λ would satisfy the problem, was based upon the application of eq. (4a) or (61) (see § 7). This means that the connection between $d\overline{U}/dy$ and $-\overline{u'v'}$ is obtained by considering the "residual" viscous friction $R^{-1}\, d\overline{U}/dy$, which must make up for the difference between the total shearing force τ and the turbulent frictional force $-\overline{u'v'}$. Now in the central region of the channel this "residual" viscous friction is a very small quantity: depending on the value of R it may be a thousand, a million times, etc. smaller than $-\overline{u'v'}$, and in most theories concerning the turbulent motion it is neglected altogether. As is well known, in the theories developed by PRANDTL and by VON KARMAN the connection between $d\overline{U}/dy$ and $-\overline{u'v'}$ is obtained by considering the transport of momentum due to the action of the relative motion, a method which seems much more logical from a physical point of view.

The difficulty signalled here may indicate a defect in the statistical considerations as far as they have been developed. Perhaps it may appear that the results obtained for the statistical distribution of the relative motion are not wholly imparted by it; it may be considered as probable besides, that the application of eq. (61) to the region very near to the walls can be justified — but the conviction presents itself that in the central region of the field the connection given by eq. (61) or (4a) is too "weak" to have an appreciable influence upon the course of $d\overline{U}/dy$, which possibly might be influenced by some other mechanism. If this would prove true, it would mean that the system of formulae assumed is not sufficient to describe the nature of the turbulent motion, and that some other conditions still must be introduced.

III. Lastly there is the problem presented by the statistical method itself. The form in which it has been used had no other justification than its apparent simplicity, and had been modeled upon the example of the statistical treatment of a conservative system. Now in Part VI we have seen that in the case of a system having an infinite number of degrees of freedom, this method leads to a divergence in the dissipation. In formal respect this result has a certain analogy in the circumstance that classical statistical mechanics leads to an infinite content of energy in the case of a conservative system with an infinite number of degrees of freedom, in equilibrium with a system having a given temperature. In the latter case a remedy is found in quantum statistics — however, it seems utterly improbable that this remedy might be used in hydrodynamics. Indeed, the general applicability of REYNOLDS' similarity law is a proof that in the case of ordinary incompressible liquids

(colloidal substances and the like being excluded) the only quantities that are relevant are the dimensions of the field (including the dimensions of the asperities of the walls, if such are present), the velocity of the flow and the density and the viscosity of the fluid; there is no indication thus far that either atomic or molecular structure, or association of molecules have a direct bearing on the phenomena observed.

The rescue must come from an other side, and the dissipative nature of the system must be of prime importance in this connection. Possibly a deeper study of relations of the nature of LIOUVILLE's theorem (comp. Part I, p. 420—423) may give some clue. Though the matter is wholly hypothetical, we shall come back to it below in § 13, after having first given some attention to an other type of problems to which the statistical method can be applied.

12. *Application of the statistical method to other systems of equations.* It is not difficult to construct certain systems of equations to which the same formal treatment can be applied, as was the case with the equations of hydrodynamics. Such a system of equations might be considered as describing some imaginary mechanical system, again of dissipative nature, but of a simpler type than fluid motion.

By way of example we may take the following one:

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= F - \varkappa v^2 + \mu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial v}{\partial t} &= + \varkappa uv + \mu \frac{\partial^2 v}{\partial y^2} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (75)$$

Here u and v are functions of the coordinate y and of the time t. We shall assume that they represent certain velocities, and it will be supposed that they are subjected to the conditions $u = 0$, $v = 0$ at the endpoints of the interval for y, which will be taken at $y = 0$, $y = \pi$ respectively. F is a constant, provisionally unknown, representing a "force", acting from the exterior upon the system and capable of doing work upon it; $\varkappa$ is a "coupling" coefficient, while μ is a coefficient of "friction". The coupling between u and v has been choosen in such a way, that it disappears from the equation of energy for the total system.

If now we introduce the condition:

$$\int_0^{\pi} u \, dy = \frac{\pi}{2} Q \quad \ldots\ldots\ldots \quad (76)$$

where Q represents a given constant, then the system can be subjected to precisely the same treatment as the system formed by the hydrodynamical equations. It will be seen that a distribution function can be

obtained of a similar nature as before; the exponent of this function can be separated into two parts, one of which governs the statistical distribution of the v-motion, while the other part governs the distribution of the u-motion. In connection with the first part a variational problem can be constructed in order to obtain a set of normal functions for the expansion of $v(y)$; the differential equation for these functions now becomes of the second order. Just as before it depends upon a certain function $\lambda(y)$, which must be adjusted afterwards; this adjustment constitutes the main difficulty of the problem from the mathematical side.

Instead of carrying out this process, however, it is more instructive for our purpose to choose another way, and to transform the system of equations by introducing a FOURIER expansion both for u and for v:

$$u = \Sigma\, A_p \sin py\,; \quad v = \Sigma\, B_k \sin ky \quad . \quad . \quad . \quad . \quad . \quad (77)$$

The coefficients A_p and B_k will be functions of the time; they form the unknowns of the transformed problem. If we put:

$$a_p = \frac{2}{\pi}\int_0^{\pi} dy \sin py \quad ; \quad a_{pkl} = \frac{2}{\pi}\int_0^{\pi} dy \sin py \sin ky \sin ly$$

(certain of these quantities will have the value zero), then the system (75) can be replaced by the following system of simultaneous differential equations:

$$\left.\begin{aligned} dA_p/dt &= a_p F - \varkappa \sum_{kl} a_{pkl} B_k B_l - \mu p^2 A_p \\ dB_k/dt &= \quad + \varkappa \sum_{pl} a_{pkl} A_p B_l - \mu k^2 B_k \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (78)$$

At the same time condition (76) takes the form:

$$\Sigma\, a_p A_p = Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

It is convenient to consider a slightly more general system, described by the equations:

$$\dot{A}_p = a_p F - a_{pkl} B_k B_l - \mu_{pq} A_q \quad . \quad . \quad . \quad . \quad . \quad (80a)$$

$$\dot{B}_k = \quad + a_{pkl} A_p B_l - \nu_{kl} B_l \quad . \quad . \quad . \quad . \quad . \quad (80b)$$

together with the condition:

$$a_p A_p = Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

For simplicity $\dot{A}_p$ has been written instead of dA_p/dt, etc., while the summation signs have been omitted, it being understood that summations must be carried out over every index occurring twice. The α_p, α_{pkl}, μ_{pq}, ν_{kl} in eqs. (80*a*/*b*) and (81) may be arbitrary numbers, satisfying the conditions of symmetry: $\alpha_{pkl} = \alpha_{plk}$, $\mu_{pq} = \mu_{qp}$, $\nu_{kl} = \nu_{lk}$, while it will be supposed that both the μ_{pq} and the ν_{kl} are the coefficients of positive definite quadratic forms. Provisionally the assumption will be introduced that the number of degrees of freedom of the system shall be *finite*: the range of the indices $p, q, k, \ldots$. etc. will be restricted to the interval from 1 to N.

Want of space prevents us from giving a detailed exposition of the working of the process described in § 2 of Part IV if applied to the present case. We mention the main point, viz. that instead of considering the full system of equations we confine ourselves to a reduced system, formed by taking all equations relating to the A-motion (eqs. (80*a*), which in this respect are equivalent to eq. (3) of § 2), while the equations relating to the B-motion are replaced by a single one, the energy equation or dissipation condition. In § 2 the latter equation was taken in the form (6), valid for the total system; the corresponding equation in the present case takes the form:

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{A_p^2 + B_k^2}{2}\right) = F\,Q - \mu_{pq}\,A_p\,A_q - \nu_{kl}\,B_k\,B_l \quad . \quad . \quad (82)$$

In order to form the exponent of the distribution function a set of numbers λ_p must be introduced, satisfying the condition: $\alpha_p\,\lambda_p = Q$; then the exponent is given by:

$$c + \beta\,[\mu_{pq}\,(\lambda_p\,A_q - A_p\,A_q) + \alpha_{pkl}\,\lambda_p\,B_k\,B_l - \nu_{kl}\,B_k\,B_l].$$

Now a linear transformation from the variables B_k to a new set of variables must be sought, which reduces the part $\alpha_{pkl}\,\lambda_p\,B_k\,B_l - \nu_{kl}\,B_k\,B_l$ of this exponent to a sum of squares. The transformation formulae of course will depend upon the λ_p. If the transformation is effected, the statistical mean values of the squares of those new variables can be calculated, and consequently also those of products of the type $B_k\,B_l$. — A similar process must be carried out with the other part of the exponent, depending upon the A's. Then also the statistical mean values of the A's can be obtained (regard must be taken of the condition (81) in performing the calculations), and further the statistical mean values of A_p^2 etc. [1])

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[1]) It will be found that $\overline{A_p^2}$ is not equal to $(\overline{A_p})^2$. This point is of importance in connection with the question raised in § 7, footnote [1]) to p. 491.

The statistical mean values now must satisfy the following relations, obtained resp. from eqs. (80a), (82):

$$a_p F - a_{pkl} \overline{B_k B_l} - \mu_{pq} \overline{A_q} = 0 \quad \text{(for every } p\text{)},$$

$$F Q - \mu_{pq} \overline{A_p A_q} - \nu_{kl} \overline{B_k B_l} = 0,$$

which together with the condition $a_p \lambda_p = Q$ form $N+2$ equations for the $N+2$ unknown quantities $\lambda_1, \ldots \lambda_N, \beta, F$.

In the general case these equations will be very complicated. However, by way of example the case $N=2$ may be taken, which can be worked out quite simply.

Apart from possible difficulties of mathematical character it thus would seem that the whole procedure runs well for a finite N. But as soon as N goes to infinity it can be shown that the expression for the total dissipation of the system again becomes divergent, and thus the same problem presents itself as before.

13. *Considerations concerning the stability of the motion of the system and the application of* LIOUVILLE*'s formula.*

The system of equations (80a/b) admits a stationary solution (independent of the time), in which all B_k are equal to zero. The equations in this case reduce to:

$$a_p F = \mu_{pq} A_q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

and a solution can be obtained in the form:

$$A_p = \beta_p F = \frac{\beta_p}{\beta_q a_q} Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

This solution may be called the "laminar motion" of the system.

The question can be raised whether this laminar motion will be stable against small disturbances? If we suppose that the values given to the B's are so small that their influence upon eqs. (80a) can be neglected, then eqs. (80b) can be written:

$$\dot{B}_k = (a_{pkl} A_p - \nu_{kl}) B_l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

with approximately constant values of the A's. If we put all B's proportional to e^{ft}, the time factor f must be a root of the determinantal equation, which in abridged form can be written (with $\delta_{kk} = 1$, and $\delta_{kl} = 0$ if $k \neq l$):

$$| a_{pkl} A_p - \nu_{kl} - f \delta_{kl} | = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

For vanishing values of Q all A_p will decrease to zero, and the determinantal equation reduces to:

$$| \nu_{kl} + f \delta_{kl} | = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

As the ν_{kl} are the coefficients of a positive definite quadratic form, all roots of this equation are negative, which means that the laminar motion is stable for sufficiently small values of Q.

On the other hand, if Q is increased without limit, the determinantal equation asymptotically approaches to the form:

$$\left| \alpha_{pkl} \frac{A_p}{Q} - \frac{f}{Q} \delta_{kl} \right| = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

and it is evident that now the sign of the roots can be changed by changing the sign of Q (which leaves unchanged the sign of A_p/Q). Hence if we assume that Q has the proper sign, by sufficiently increasing Q it is possible to make one or more of the roots of the determinantal equation positive. The laminar motion then becomes unstable, and "turbulence" sets in.

Now a second question can be raised. Suppose that the value of Q surpasses the stability limit, and that turbulent motion has appeared; will this turbulent motion affect all degrees of freedom of the system, or only a certain number of them — depending upon the magnitude of Q? I would believe that this question cannot be answered by simply counting how many roots of the determinantal equation are positive. As the A's and B's are coupled in a non linear way, the actual motion of the B's is not a simple superposition of mutually independent modes of motion, of which those that have negative time factors are soon damped out never to appear again. On the contrary: in consequence of the disturbances which are due to the variation of the A's it is possible that certain modes of motion of the damped type will have a chance of being continually excited anew. On the other hand modes of motion with positive time factors will be disturbed in such a way, that they cannot increase beyond all limits (the coupling between the B-motion and the A-motion acts in a similar way as the coupling between relative motion and principal motion in the hydrodynamical case).

We thus must look for an other criterion.

Let us imagine a multi-dimensional space, with coordinates $B_1, B_2, \ldots B_N$. A point in this space will represent the state or "phase" of the B-motion at a given instant. A set of points will represent an assembly of such instantaneous states. As the B's are functions of the time, the set of points will describe a certain motion through the multi-dimensional space. Now the property to which LIOUVILLE's theorem — in the present case we may better speak of LIOUVILLE's formula — refers, is the variation of the density of this set of points in course of time. If we exclude the possibility that the whole set of points moves towards infinity in some special direction, then in the case of a density which is independent of the time we may conclude that an assembly of sufficient extension will possess a certain statistical stability. On the contrary, if the density

should appear to be increasing, it would prove that the points are drawing together, and thus that there is a preference for certain modes of motion, while others are gradually disappearing from the assembly.

The variation of the density with respect to time of a group of points moving through a given region of the multi-dimensional "phase space", is determined by the equation:

$$\frac{\partial \varrho}{\partial t} = -\frac{\partial \dot{B}_k}{\partial B_k} = -\alpha_{pkk} A_p + \nu_{kk}. \quad . \quad . \quad . \quad . \quad . \quad (89)$$

As the A's are functions of the time, an exact discussion of the variation of the density will be a difficult matter. However, in order to fix the ideas, let us consider the average value, obtained by inserting the statistical mean values of the A's.

If first we return to the case of very small values of Q, in which case all A's are small too, the expression (89) certainly will be positive, as all ν_{kk} must be positive. In this case the set of points will be crowding together, and will finally contract into the origin of the coordinate system (all B_k becoming zero). But as soon as Q surpasses the limit of stability of the laminar motion, this no longer will be the case. Yet the fact that Q surpasses the stability limit does not imply that the expression (89) (or rather its statistical mean value) immediately ceases to be positive, and examples can be constructed where this is not the case [1]). In such a case there appears to be a tendency towards condensation, which, however, cannot result in a contraction of the whole set into the origin.

This brings us to the supposition that the set of points perhaps may condense into a subspace of a smaller number of dimensions than the original phase space of N dimensions.

Let us therefore consider the projection of the motion of the set of points upon a subspace of, say, s dimensions. Then the density ϱ_s of the set of projected points in this subspace can be defined, and it will be possible to deduce an expression for $\partial \varrho_s / \partial t$. It can be taken for certain that there will be some projections (perhaps upon subspaces of a relatively small number of dimensions), for which the average value of $\partial \varrho_s / \partial t$ will be negative, while for other projections (with s only slightly smaller than N) it will be positive.

We now might venture the hypothesis that we must look for the most comprehensive subspace (i.e. of the largest number of dimensions) for which the average value of $\partial \varrho_s / \partial t$ is not yet positive — preferably it must be as near to zero as possible. The motion of the projection of the set of points upon this "maximum subspace" (as we may call it) then

[1]) The expression occurring on the right hand side of eq. (89) is connected with the sum of the roots of eq. (86), provided the values of the A's are the same in both equations. This point may have some importance for our problem, but as in general the A's are functions of the time the actual relations will be far more complicated.

may be considered as being statistically stable, while the extent of the set in directions not belonging to this subspace ultimately will diminish to zero. If S is the number of dimensions of the "maximum subspace", then it may be supposed that the turbulent motion on the average will be confined to the S degrees of freedom, corresponding to the coordinates that subtend the "maximum subspace", and that *the application of the statistical method must be restricted to these degrees of freedom only.* The justification of the latter proposition is found in the circumstance that the density of the points in the projection upon the "maximum subspace" upon the average is independent of the time, so that, if this subspace is divided into cells of equal volume, the same weight must be given to every cell.

The number S probably will increase with the value of Q. In the case of a system having a finite number of degrees of freedom perhaps difficulties may arise, as soon as S approaches to N (the motion perhaps might become wholly divergent). But in the case of an infinite number of degrees of freedom, there would be no objection to an unlimited increase of S, which simply would mean that more and more degrees of freedom come into play if Q is increased continuously. And this would fit in very well with our original problem.

All this has been expressed in a crude and very primitive way, and the deduction of a more satisfactory formulation may prove a matter of great difficulty. The restriction of the statistics to a subspace of a definite number of dimensions has something arbitrary and schematical in its character; it may be that the limit introduced in this way in reality is more of the nature of an average. — There is also the difficulty with the A's, which are variables themselves (it is only provisionally that we have inserted the statistical mean values). Perhaps it will be necessary to consider a multi-dimensional space in which both the A-motion and the B-motion can be represented; the formulation of the hypothesis then must be altered.

Still the point to which I would draw the attention is that the investigation of the variations of the density of the projections of a set of representative points upon various subspaces may afford us a means either for restricting the number of degrees of freedom to be considered in the statistics of turbulent motion, or perhaps for the introduction of a certain weight function influencing the distribution function in such a way, that the statistical frequency of certain modes of motion is diminished so much, that the expression for the total dissipation will become convergent. If this should prove to be the case, then the difficulty upon which we struck in Part VI might be overcome.

Chemistry. — *Osmosis in systems consisting of water and tartaric acid and containing three liquids, separated by two membranes.* II. By F. A. H. SCHREINEMAKERS and H. H. SCHREINEMACHERS.

(Communicated at the meeting of June 24, 1933).

V. *Influence of the position of the two membranes on the stationary state.*

If for the sake of simplicity we substitute the 3.999 % of tartaric acid of system (22) in Comm. I by 4 %, we have the system:

$$\begin{array}{c} \textit{cell.} \quad \triangle m > o \quad \textit{bl.} \\ \textit{inv.}\,(W) \,|\, \textit{stat.}\,(\pm\, 1.958\ {}^0/_0) \,|\, \textit{inv.}\,L'\,(4\ {}^0/_0\ \textit{tart. ac.}) \quad . \quad . \quad . \quad (1) \\ \rightarrow \qquad W \qquad \leftarrow o * \\ \leftarrow \quad \textit{tart. ac.} \quad \leftarrow + \end{array}$$

in which the membrane on the left consists of cellophane and the membrane on the right of pig's bladder. As appears from the arrows, the water diffuses inwards through both membranes; the tartaric acid diffuses inwards through the bladder and outwards through the cellophane. As has been indicated by the sign + with the corresponding arrow, more tartaric acid diffuses inwards than outwards, however. The quantity of the stat. liquid continuously increases, as has been indicated by the sign $\triangle m > o$.

If we also substitute the 4.005 % of tartaric acid of system (25) in Comm. I by 4 %, we have the system:

$$\begin{array}{c} \textit{bl.} \quad \triangle m < o \quad \textit{cell.} \\ \textit{inv.}\,(W) \,|\, \textit{stat.}\,(\pm\, 1.936\ {}^0/_0) \,|\, \textit{inv.}\,L'\,(4\ {}^0/_0\ \textit{tart. ac.}) \quad . \quad . \quad . \quad (2) \\ \leftarrow o * \qquad W \qquad \rightarrow \\ \leftarrow + \quad \textit{tart. ac.} \quad \leftarrow \end{array}$$

in which the membrane on the left now consists of a bladder and the membrane on the right of cellophane; the two invariant liquids, however, are the same as in system (1).

The water, however, now diffuses outwards through the two membranes; the tartaric acid, however, still diffuses towards the left through both membranes just as in (1), but now, as has been indicated by the sign +

with the corresponding arrow, more tartaric acid runs outwards than inwards. The quantity of this stationary liquid now decreases continuously, as has been indicated by $\triangle m < o$, whereas in (1) it increased continuously.

In the systems (1) and (2) one of the invariant liquids consisted of pure water; the osmosis (namely the direction of the arrows etc.) will not change, however, when the water contains a little tartaric acid besides, we shall call this liquid "diluted" and we shall represent it by L_d. The same obtains also when the other invariant liquid contains some little quantity of tartaric acid; as we assume, however, that this liquid contains more tart. acid than L_d, we shall call it "concentrated" and we shall represent it by L_c.

We now represent these systems by:

$$\begin{array}{c} \textit{cell.} \qquad\qquad \textit{bl.} \\ \textit{inv.}\ L_d \mid \triangle\ m > o \mid \textit{inv.}\ L_c \\ \rightarrow \quad W \quad \leftarrow o * \\ \leftarrow \textit{tart. ac.} \leftarrow + \end{array} \qquad \ldots\ldots\ldots \quad (1a)$$

$$\begin{array}{c} \textit{bl.} \qquad\qquad \textit{cell.} \\ \textit{inv.}\ L_d \mid \triangle\ m < o \mid \textit{inv.}\ L_c \\ \leftarrow o * \quad W \quad \rightarrow \\ \leftarrow + \textit{tart. ac.} \leftarrow \end{array} \qquad \ldots\ldots\ldots \quad (2a)$$

which we shall call (1*a*) and (2*a*).

From this follows among other things:

1. When the diluted liquid L_d is next to the cellophane (system 1*a*), the quantity of the stationary liquid increases continuously; when the diluted liquid L_d is next to the bladder, the quantity of the stationary liquid decreases continuously (system 2*a*).

2. If we change the places of the two membranes of one of the systems (1*a*) or (2*a*), one system will pass into the other. From this it appears that the interchange of the membranes has a great influence on the progress of the osmosis (comp. 1 and 1*a* with 2 and 2*a*).

3. If in system (1*a*) we add water on the left side and a little tartaric acid on the right side, the left-side liquid will remain diluted and the right-side liquid concentrated; the osmosis will then develop in a similar way.

4. If in system (1*a*) we add a sufficient quantity of tartaric acid on the left side, and on the right side a sufficient quantity of water, so that the left-side liquid now becomes the concentrated- and the right-side liquid the diluted one, (1*a*) will pass into the system:

$$\begin{array}{c} \textit{cell} \qquad\qquad \textit{bl.} \\ \textit{inv.}\ L'_c \mid \triangle\ m < o \mid \textit{inv.}\ L'_d \end{array} \qquad \ldots\ldots\ldots \quad (3)$$

As the diluted liquid now is next to the bladder, this system now is the same as (2*a*), only the sequence from left to right is different.

From 3 and 4 it now appears that for the progress of the osmosis in system (1*a*) it makes a great difference whether we add water on the left side and tartaric acid on the right side or vice versa. Of course the same also obtains for system (2*a*).

We may suppose the liquids i_1 and i_2 of the stationary system:

$$inv.\, i_1 \,|\, stat.\, L \,|\, inv.\, i_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

remain invariant, because of a continuous current of i_1 and i_2 flowing along the membranes, or because i_1 and i_2 are a kind of tissues in which the state remains practically invariant. In the space between these two invariant states, a stationary liquid L will then form.

We now imagine the space of the stationary liquid completely closed, but for a tiny opening, and the quantity of this liquid continuously increasing; we may then compare this space with the cavity of a gland and liquid L with the fluid secreted.

This secreted fluid generally has quite an other composition than the currents of tissues i_1 and i_2; nor can it generally be looked upon as a mixture of these liquids.

In this simple example the composition of this glandular secretion depends upon:

1. the state of the two currents or tissues i_1 and i_2.

2. the nature of the two membranes and the ratio of their surfaces. Every change in one or more of these factors will also cause a smaller or greater change in the composition of the glandular secretion.

We now may suppose also that one or two membranes gradually change in such a way that the quantity of the secretion, instead of increasing continuously, now begins to decrease. The gland will then stop functioning as such, although the two tissues may have remained perfectly unchanged: of course this may also be the case when one or both tissues change gradually [1]).

VI. *A combination of two membranes.*

If we place a piece of cellophane and a piece of a pig's bladder against each other in such a way that they stick closely together and no room is left between them, then we may consider this combination as a single membrane, the one layer consisting of cellophane and the other of a blad-

[1]) Later on one of us will refer in extenso to stationary states in connection with tissues and glands. Comp. also F. A. H. SCHREINEMAKERS, These Proceedings 35, p. 1131 (1932).

der. If we now bring this combination-membrane between the two liquids L_d and L_c, we have the osmotic systems:

$$\begin{array}{r|l} \textit{cell.} & \textit{bl.} \\ \textit{inv.}\ L_d & \textit{inv.}\ L_c \end{array} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1b)$$

$$\begin{array}{r|l} \textit{bl.} & \textit{cell.} \\ \textit{inv.}\ L_d & \textit{inv.}\ L_c \end{array} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2b)$$

which we shall call (1*b*) and (2*b*). These systems now consist of the same liquids and the same membrane, which however in (1*b*) turns its cellophane-layer towards the left and in (2*b*) towards the right. We get these systems (1*b*) and (2*b*) by placing the two membranes in (1*a*) and (2*a*) against each other.

As osmosis occurs in these systems (1*b*) and (2*b*), the water and the tartaric acid will diffuse continuously in some direction or other through the two layers and consequently also through the boundary plane they have in common.

We now can imagine that at a certain moment of the osmosis the cellophane and the bladder under some influence or other will get loose from each other in some small spot, so that a small cavity will form between the two layers; this will then be filled with a small quantity of the diffusing substances and will pass into a stationary state. We now distinguish two cases.

1. When this takes place in system (1*b*), it passes into system (1*a*) in which, however, there is only a small quantity of the stationary liquid as yet. As this quantity continuously increases, however, the two layers will get further away from each other. We now represent this system by:

$$\overset{\textit{cell.}}{\textit{inv.}\ L_d}\left(+\right)\overset{\textit{bl.}}{\textit{inv.}\ L_c} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

in which for the sake of clearness a biconvex form has been given to the combination.

2. When it takes place in system (2*b*), viz. the cellophane and the bladder getting loose in a small spot, (2*b*) will pass into (2*a*). As the quantity of the stationary liquid now decreases, however, the small quantity of liquid that has penetrated between the two layers, will disappear again; so the membrane will not become biconvex now, but remain unchanged.

To be able to observe these phenomena, we took the system

$$\begin{array}{r|l} \textit{cell.} & \textit{bl.} \\ \textit{Water} & L\ (4\ \%\ \textit{tart. ac.}) \end{array} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

in which, however, the two liquids were not kept invariant, so that they

changed during the osmosis; this does not influence the occurring or non-occurring of these phenomena, however.

After some hours (6) had passed already into a system

$$\overset{cell.}{L_d}\left(+\right)\overset{bl.}{L_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which the liquid between the two layers could already be seen quite clearly, its quantity increasing continuously.

When this combination-membrane was turned round[1]), and we consequently had the system

$$\overset{bl.}{L_d}\left(-\right)\overset{cell.}{L_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

we saw the liquid absorbed disappear again and the membrane becoming flat; this was still the case when we again replaced L_d by pure water and L_c again by a liquid with 4 % of tartaric acid.

It appears from these investigations and theoretical considerations that similar phenomena are to be expected with all membranes, consisting of two or more layers of a different nature; the turning of the membrane or, what is the same, the interchange of the two liquids may here have a greater or a smaller influence on the progress of the osmosis, depending on the difference in the nature of the layers and the compositions of the liquids.

In order to show the influence of the position of a membrane (or of its turning) in still another way we took the two systems:

$$\begin{array}{r|l} cell. & bl. \\ L\,(2\,\%) & L\,(2\,\%), \end{array} \quad (1c) \qquad \text{and} \qquad \begin{array}{r|l} bl. & cell. \\ L\,(2\,\%) & L\,(2\,\%) \end{array} \quad . \quad . \quad (2c)$$

which we shall call (1c) and (2c). As the two liquids had the same composition in both systems (namely 2 % of tart. acid) there was no osmosis of course. The two systems differ only in the position of the membrane. We now added in both systems water on the left side and a little tartaric acid on the right side.

Then system (1c) passed into a system (7), in which after a few hours a liquid could already clearly be observed between the cellophane and the bladder, its quantity continuously increasing.

In system (2c) no liquid formed between the two layers of the combination-membrane.

When we turned the membrane in system (7) which had developed from system (1c), we saw, as was indeed to be expected, that the liquid taken in between the two layers, disappeared again.

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[1]) Experimentally it is simpler of course not to turn the membrane, but to interchange the two liquids; indeed we did it in this way.

If in the osmotic system

$$inv.\,(Water)\,|\,L\,(W + tart.\,ac.) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

we bring a membrane of cellophane, the osmosis proceeds according to the D.T.

$$\rightarrow W \qquad \leftarrow tart.\,ac. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

no matter what concentration the variable liquid L may have at the beginning of the osmosis. (Comm. I, pg. 1243).

If, however, we bring a membrane of pig's bladder in this system, then the osmosis does depend upon the concentration the variable liquid has at the beginning of the osmosis, and no water will diffuse when liquid L has a certain concentration; we have called this liquid the neutral liquid L_n (Comm. I, pg. 1243).

If at the beginning of the osmosis we now take for L in system (9) a liquid with a greater amount of tart. acid than liq. L_n, then the substances will first diffuse for some time according to the first D.T. and afterwards until the end of the osmosis according to the second D.T. of (11) namely:

$$\left.\begin{array}{lll} first & \rightarrow W & \leftarrow tart.\,ac. \\ later\ on & \leftarrow o * W & \leftarrow tart.\,ac. \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The composition of the neutral liquid L_n at which the water-diffusion changes its direction, depends upon the nature of the pig's bladder used; with one of the bladders we found $\pm$ 14 %, with an other $\pm$ 17 % and with a third $\pm$ 21 % of tart. acid.

We now may put the question according to which D.T.'s the osmosis will proceed, when we bring a combination-membrane of cellophane and a pig's bladder in system (9).

In order to examine this we took the system

$$\begin{array}{rl} bl & cell. \\ inv.\,(Water) & L\,(beg.\ 28\ 1\ {}^{0}/_{0}\ tart.\,ac.) \end{array} \quad . \quad . \quad . \quad . \quad (12)$$

in which the variable liquid contained 28.1 % of tart. acid at the beginning of the osmosis. As the combination-membrane is next to the water with its layer of bladder, it was very probable that the combination would behave during the entire osmosis as a single membrane, namely that no liquid would form between the two layers (comp. 2*b* and 8); the experiment has indeed confirmed this.

The data for this osmosis are found in table 1. In column 1 we find the numbers of the successive determinations, in column 2 the number of hours that passed after the beginning of the osmosis; in column 3 we find the amount of tart. acid of the variable liquid. In columns 4 and 5 we find the number of grams of tart. acid and water, taken in (sign +) or given off

TABLE I

N°.	t in hours	% tart. ac. of the variable liq.	Diffused to the var. liq.		$\triangle m$
			gr. tart. ac.	gr. W	
1	0	28.10			
2	24	27.71	— 0.98	+ 2.728	+ 1.748
3	113	26.02	— 4.53	+ 10.456	+ 5.926
4	237	24.35	— 4.29	+ 10.492	+ 6.202
5	424	22.12	— 6.54	+ 12.266	+ 5.726
6	664	20.10	— 5.94	+ 11.038	+ 5.098
7	1024	17.69	— 7.52	+ 12.222	+ 4.702
8	1433	15.55	— 7.03	+ 9.386	+ 2.356
9	1907	13.51	— 6.97	+ 7.042	+ 0.072
10	2431	11.60	— 6.40	+ 5.945	— 0.455
11	3049	9.346	— 7.36	— 1.010	— 8.370
12	3528	7.345	— 6.10	— 1.973	— 8.073
13	4166	4.632	— 7.61	— 5.087	— 12.697

(sign —) by the variable liquid between two successive determinations. From these columns 4 and 5 the last column follows at once, namely the total quantity $\triangle m$ taken in (sign +) or given off (sign —) by the variable liquid between two successive determinations.

From this it appears that the tartaric acid diffuses ← during the entire osmosis and the water first → and afterwards ← o ⋆ so that the two D.T.'s (11) occur; the neutral liquid contains ± 10 % of tart. acid.

So this combination-membrane of a bladder and cellophane always behave as if it consisted of a bladder only; yet there are differences of course; the velocity of the osmosis namely is much smaller (after 4166 hours or almost 173 days the amount of tartaric acid, which at the beginning of the osmosis was 28.1 %, had decreased to no more than 4.632 %, whereas this amount in a system with a bladder only, had decreased after more than 34 days from 25.768 % to 0.137 % already); the neutral liquid also has a smaller amount of tartaric acid (namely ± 10 %) than when only a bladder was used (namely ± 14, 17 and 21 %).

Leiden, Lab. of Inorg. Chemistry.

Chemistry. — *The Allotropism of Beryllium*. Bij F. M. JAEGER and J. E. ZANSTRA.

(Communicated at the meeting of June 24, 1933).

§ 1. In a previous paper [1]), in which the remarkable behaviour of crystalline lumps of *beryllium* was described, when the metal is heated above a certain temperature, we already mentioned that from the experiments hitherto made, it must be concluded that some allotropic change in the metal seems to take place, although evidently no *direct* connection between this phenomenon and the abnormal calorimetric behaviour exists. In a following publication we intend to return to these calorimetric phenomena, since it was discovered that also samples of this metal occur that do *not* manifest the said behaviour; while, on the other hand, we observed the phenomena described also in the case of some other metals, in the first place with *zirconium*. In the present paper we only wish to publish the results of the investigation of the metal by means of *X*-rays, which has led us to the discovery of a new modification of *beryllium*, —this being probably a metastable form of the metal with respect to the ordinary *α-beryllium*.

The structure of *beryllium* has been determined by G. MEIER and by Mc KEEHAN[2]); according to the latter, *α-beryllium* has a closest-packed hexagonal arrangement, with $a_0 = 2{,}283$ Å. and $c_0 = 3{,}61$ Å. and with two atoms of *Be* within its simple parallelopidedic cell; $d_0 = 1{,}827$. There is, however, no complete agreement between the observed and the calculated intensities of the different reflecting forms; moreover, the author already mentions the occurrence of a number of unexplained lines in his spectrograms, whose presence he partially attributes to the admixture of a small quantity of beryllium-oxide, and partially tries to account for by assuming the presence of an unknown substance.

At our disposal we had two very pure samples of *beryllium* [1]): the one in the form of very beautiful crystal-aggregates, the other in the shape of a massive cylindrical rod, which inter alia was used by E. J. LEWIS [3]) in his experiments on the electrical conductivity of the metal and which most kindly was put at our disposal by Prof. E. MEERRITT of *Cornell*-University. The results obtained with these two samples are

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[1]) F. M. JAEGER and J. E. ROSENBOHM, These Proceedings Amsterdam, **35**, (1932), 1055; conf. p. 1061.

[2]) G. MEIER, Thesis Göttingen, (1921); L. W. Mc KEEHAN, Proc. Nat. Acad. of Science, **8**, (1922), 270; M. C. NEUBURGER, Zeits. f. Kryst., (A), **85**, (1933), 325.

[3]) E. J. LEWIS, Phys. Review, **34**, (1929), 1577.

practically identical, so that in the following no special distinction between them is made.

§ 2. *α-Beryllium.*

From the sample of pure, well crystallized *beryllium*, provided by SIEMENS, some single crystals, showing the face $\{0001\}$ and some others, showing a hexagonal prism, could be isolated. Rotationspectrograms were made with the *c*-axis and the orthohexagonal *b*-axis, respectively, as axes of rotation. Also oscillation-spectrograms on $\{0001\}$, after BRAGG's method, were prepared, — calcite being used as a standard-material. The analysis of the rotationspectrograms was made by means of BERNAL's method.

A. *Rotation-spectrogram round the c-axis.*

Copper-α-radiation; 50 m. Amp. hours. The spectrogram showed a principal and the first accessory spectrum.[1])

Principal spectrum: $(110) = (01\bar{1}0)$; $(310) = (11\bar{2}0)$.

1st Access. Spectrum: $(111) = (01\bar{1}1)$. $I_c = 3{,}64$ Å (corrected; see C).

No abnormal reflections were observed with these crystals.

B. *Rotation-spectrogram round the orthohexagonal b-axis.*

Principal spectrum: $(200) = (10\bar{1}0)$; $(002) = (0002)$; and $(202) = (10\bar{1}2)$.

1st Access. Spectrum: $(111) = (01\bar{1}1)$; and $(112) = (01\bar{1}2)$. $I_b = 2{,}28$ Å.

No abnormal reflections were found.

C. An oscillation spectrogram on $\{0001\}$ was obtained by means of *molybdenum*-radiation; a second order reflection was found, calcite being used for the purpose of comparison. From the observed value: $d_{(0001)} = 1{,}82$ Å., I_c is calculated to be: 3,64 Å.

The axial ratio of *α-beryllium* is, therefore, calculated at: $a:c = 1:1{,}5877$.

As the specific weight of the metal is about 1,83, the single parallelopipedic cell contains *two* atoms of beryllium; $d_{calc.} = 1.816$.

§ 3. Although many of the powderspectrograms prepared showed, — as was already mentioned, — a number of extra-diffraction-lines, some of them, however, proved to be completely free from those.

Thus a powderspectrogram, prepared with *Cu*-radiation ($R = 57{,}2$ m.M.; exposure = 32 m. Amp. hours) showed the following lines (*hkl*), as calculated from the angles θ (see Table I).

The values of $\sin^2\theta$ are calculated from the formula:

$$\sin^2\theta = 0{,}1515\,(h^2 + hk + k^2) + 0{,}0453\,.\,l^2.$$

Of the lines (100) and (101) also β-images were occasionally observed.

[1]) The orthohexagonal indices being $(h_0\,k_0\,l_0)$ and the hexagonal ones (BRAVAIS) being $(h\,k\,\bar{i}\,l)$, we have the relations: $h_0 = 2h + k$; $k_0 = k$; $l_0 = l$ and consequently: $2h = h_0 - k_0$; $k = k_0$; $2\bar{i} = h_0 + k_0$ and $l = l_0$. The orthohexagonal axes are: $a_0\sqrt{3}$; a_0; and c_0.

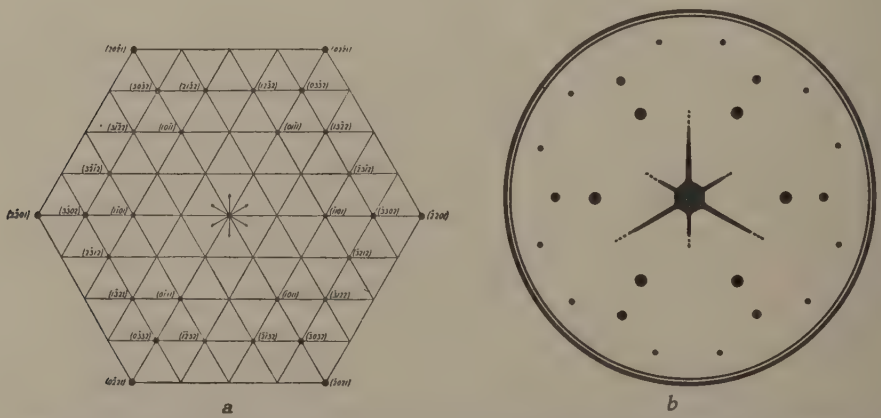
TABLE I.

Powderspectrogram of α-Beryllium.

Int.:	Θ:	$\sin^2\Theta$: (observ.)	$\sin^2\Theta$: (calcul.)	$(h\,k\,l)$ (BRAVAIS):
7	22° 15′	0.1516	0.1515	$(100) = (10\bar{1}0)$
6	25 32	0.1856	0.1812	$(002) = (0002)$
10	26 30	0.1986	0.1968	$(101) = (10\bar{1}1)$
6	35 34	0.3378	0.3327	$(102) = (10\bar{1}2)$
6	42 35	0.4556	0.4544	$(110) = (11\bar{2}0)$
4	48 40	0.5696	0.5593	$(103) = (10\bar{1}3)$
3	51 33	0.6131	0.6060	$(200) = (20\bar{2}0)$
6	53 20	0.6417	0.6358	$(112) = (11\bar{2}2)$
5	54 10	0.6551	0.6516	$(201) = (20\bar{2}1)$
4	63 14	0.7972	0.7875	$(202) = (20\bar{2}2)$
3	69 8	0.8731	0.8765	$(104) = (10\bar{1}4)$

§ 4. From these data it must be concluded, that *α-beryllium* has, indeed, the structure indicated by Mc KEEHAN, — the parameter in the direction of the *c*-axis being $\frac{1}{2}\,c_0$. However, the calculated and observed intensities do *not* agree in all respects: thus, for instance, the calculated intensity of (100) is smaller than that of (002). But it must be emphasized that the beryllium-crystals used showed a most distinct *mosaic*-structure; and also other pecularities of the crystals: e. g. the existence of cleavage- and glidingplanes, — are not taken into account in the calculations of the intensities.

The LAUE-pattern on $\{0001\}$, obtained by means of a *tungsten*-anticathode showed a hexagonal symmetry, but simultaneously a remarkable *asterism*.



a *b*

Fig. 1. Gnomonic Projection (*a*) of the LAUE-pattern (*b*) of Beryllium on $\{0001\}$, obtained with *Wo*-radiation.

As the voltage used was 40 K.V., the maximum wave-length is about 0,9 Å, the minimum wave-length perhaps 0,3 Å. The most intensive spot $(22\bar{0}1)$ corresponds to: $\lambda = 0{,}461$ A; the other spots are: $(330\bar{2})$, $\lambda = 0{,}4165$ Å; $(23\bar{1}2)$, $\lambda = 0{,}3995$ Å and $(22\bar{0}2)$, $\lambda = 0{,}3415$ Å.

The asterism observed might perhaps indicate that a *ditrigonal* symmetry D_{3d}^3 is really present; in that case the two *Be*-atoms in the cell would have the positions: $[\frac{1}{2}\ {}^2/_3\ u]$ and $[{}^2/_3\ {}^1/_3\ u]$, in which u is equal to: 0,25.

§ 5. *β-Beryllium.* In a considerable number of cases the powder-spectrograms of *beryllium* prove to contain a set of accessory lines, which cannot be inserted within the normal spectrum of the *α-beryllium*. Such extra-lines were also observed by other investigators; they were attributed either to an admixture of beryllium-oxide or to "unknown" impurities. However, the relative intensities of these accessory lines with respect to the others, made it highly improbable, that they would correspond to the oxide; many of them did, moreover, not coincide with the diffraction-lines of the oxide, which was studied by using the same radiation (*iron*) as that used in the study of the metal itself [1]).

One of the lines observed by us was the same as that formerly attributed by Mc KEEHAN to an "unknown" element *X*; it was met with as well in the case of the American, as in that of the German sample. However, it soon appeared more probable that these extra-lines would correspond to another modification of the metal, the existence of which was previously suspected by us [2]) for other reasons.

Preliminary investigations had taught us that the extra-lines appeared most distinctly after a prolonged heating of the metal above 500° or 700° C.; the most favorable results were obtained by heating at 630° C. in a perfect vacuum for 10 hours. The same crystals which were used in the former experiments at ordinary temperature, — they, therefore, yielding *no* other lines than the normal ones mentioned above, — were embedded in a layer of carefully calcined magnesia, contained in a small tube (0,1 cm.) of pyrex-glass; during the heating the tube was evacuated, the pressure remaining under about 0,1 mm. of mercury. By this treatment the crystals proved to be changed in no way; they preserved their original lustre and yielded the same sharp reflections as before, while the glass-tube was neither attacked by the metal, — this being isolated from the walls of the tube by the protecting layer of magnesia.

A'. *Rotation-spectrogram round the c-axis.* [3]).

The most intensive spots of this spectrogram were the same as those

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[1]) Berylliumoxide is also hexagonal: $a_0 = 2.694$ Å.; $c_0 = 4.392$ Å. There are 2 molecules in the elementary cell.

[2]) F. M. JAEGER and E. ROSENBOHM, These Proceed. 35, (1932), 1055.

[3]) Some of these spectrograms were prepared in this laboratory by Mr. BEINTEMA.

of the normal, not heated crystal, mentioned sub A. Besides these, however, a number of *new* spots proved to be present, which are rigorously orientated with respect to the original ones. They were arranged in a principal spectrum and the 1st, 2nd, 3d and 5th accessory spectra:

Principal spectrum: *(200) or *(110); *(310); (400); (420); (600).

1st *Access. spectrum*: (201) or (111).

2nd *Access. spectrum*: *(202) or *(112); (402).

3d *Access. spectrum*: (203) or (113); *(313); (423); (623).

5th *Acc. spectrum*: *(205) or (115).

The indices mentioned correspond to orthohexagonal axes, deduced from the row-lines, with $a_0 = 7{,}1$ Å and $b_0 = 4{,}1$ Å; while the identity-distance I_c is 10,8 Å, i. e. $3 \times c_0$ of the α-modification.

The 1st spectrum is weak, the 4th is absent, while the 3d and 5th spectra are rather intensive; the directions of the c_0-axes of the two forms coincide. Evidently the orthohexagonal lattice is centred in its basal plane, as triplets with $(h + k) = odd$, do not occur.

B'. *Rotationspectrograms round the b-axis* (= 4,1 Å).

The four spectrograms prepared all showed two or three spectra; the orthohexagonal indices of the spots observed were:

I. *Principal spectrum*: *(200); (400); (402); (201); (202); *(203).
1st *Access. spectrum*: *(111); (311); *(111); *(112); (313).
2nd *Access. spectrum*: (222).

II. *Principal spectrum*: *(002); *(003); (005); (205); (203); (205).
1st *Access spectrum*: *(115); *(112); (113); (313).
2nd *Access spectrum*: (221); (220).

III. *Principal spectrum*: *(202); (203); (402); (205).
1st *Access spectrum*: (111); (313); (112); (113); *(115).
2nd *Access spectrum*: (221); (220).

IV. *Principal spectrum*: (203); (006); (002); (003); (005).
1st *Acces spectrum*: *(112); (113); (313); (113); *(115).

All occurring spots are accounted for by means of an orthohexagonal elementary parallelopipedic cell of the dimensions:

$$I_a = 7{,}1 \text{ Å}; \quad I_b = 4{,}1 \text{ Å}; \quad I_c = 10{,}8 \text{ Å}.$$

§ 6. In this way it now became possible to find the indices-triplets of the exra-lines observed by us in the different powder-spectrograms and and also those already observed by Mc. KEEHAN. This investigator used a *molybdenum*-anticathode, whose β-radiation was considerably weakened by selective absorption. The observed three abnormal lines, whose $sin^2\theta$, — calculated for *Fe*-radiation, — corresponded to 0,1945; 0,2179 and 0,0977. The two first he attributed to the presence of *beryllium-oxyde*; but this is highly improbable, because their intensities here are much too great, considering the minimal traces of the oxide present in our preparations. Moreover, the line with $sin^2\theta = 0{,}1722$ (*Fe*-radiation) in the spectrum of *Be* 0 has the *same* intensity as that with $sin^2\theta = 0{,}2179$

($=0{,}2208$); so that it is inconceivable, why the reflection with $sin^2\theta = 0{,}1722$ does *not* equally well appear amongst the extra-lines. The line with $sin^2\theta = 0{,}1945$ might correspond to $\beta\,(100)$ of *α-beryllium*; but the observed intensity ($= 6$) is then much too great in comparison with that of the corresponding α-line. The third line, with $sin^2\theta = 0{,}0977$ is the one, attributed by Mc. KEEHAN to an "unknown substance *X*"; it was, however, constantly observed by us in *all* powderspectrograms of *beryllium*, which showed such extra-lines. As chemical analysis proved that the two samples of *beryllium* used by us contained at least 99,5 % *Be* and only traces of *Fe*, while *no* iron-lines were ever observed, — it is most probable that the reflections observed are really characteristic of *beryllium* itself. Moreover, the intensities of the extra-lines are still greater, if a previously heated *beryllium*-crystal be taken as an object and, if then after powdering, a powderspectogram is made.

The coefficients of the quadratic equations, derived from the values: $a_0 = 4{,}1$ Å and $c_0 = 10{,}8$ Å and calculated for Cu_α- and Cu_β-radiation, are:

$$sin^2\theta = 0{,}0470.\ (h^2 + hk + k^2) + 0{,}00505.\ l^2 \quad (\alpha\text{-radiation}), \text{ and}$$
$$sin^2\theta = 0{,}0382.\ (h^2 + hk + k^2) + 0{,}00409.\ l^2 \quad (\beta\text{-radiation}).$$

The occuring extra-lines with $sin^2\theta = 0{,}0349$; $0{,}0621$; $0{,}0840$ and $0{,}1135$ can, however, *not* correspond to these equations; if, however, the other possible value of a_0, i. e. 7,1 Å, be taken, these quadratic equations become:

$$sin^2\theta = 0{,}0157.\ (h^2 + hk + k^2) + 0{,}00505.\ l^2 \quad (\alpha\text{-radiation}) \text{ and}$$
$$sin^2\theta = 0{,}0127.\ (h^2 + hk + k^2) + 0{,}00406.\ l^2 \quad (\beta\text{-radiation});$$

and now the interpretation of the diffraction-lines mentioned no longer presents any real difficulty.

Two samples of *beryllium*, previously heated for 10 hours at 650° C. and for 20 hours at 800° C. respectively, yielded, — besides the normal lines of *α-beryllium*, — the following extra-lines ($R = 44.4$ mm.; exposure: 50-56 m. Amp. hours).

The intensities (see Table II on following page) were estimated relatively to that of (101) of the α-modification, this intensity being taken $= 10$. To the lines having greater values of $sin^2\theta$, many possible indices-triplets can be attributed; so that these symbols become rather arbitrary.

A powdered sample of ordinary, not previously heated, metal was studied by means of *iron*-radiation (70 m. Amp. hours); the rather strong secondary radiation in the camera was prevented by screening off with a very thin aluminium-foil. The spectrogram (see Table III on following page) showed the following extra-lines, — all corresponding with the lines for *copper*-radiation already mentioned and indicated by *; the lines indicated by ** were those also observed by Mc. KEEHAN, who in his experiments used a *molybdenum*-anticathode.

TABLE II.
Diffractionlines of β-Beryllium.
(Copper-radiation).

Int.:	Θ:	$sin^2\,\Theta$ (observed):	Indices $\{h\,k\,l\}$ calculated from the quadratic formula:
3	10° 46′	0.0349**	(102)
2	11 56	0.0428*	(110) and (003)
3	13 25	0.0538*	(111)
2	15 6	0.0679**	(112)
2	16 10	0.0755*	(113)
1	16 56	0.0848*	(202)
2	17 51	0.0940*	(113)
3	19 51	0.1087*	(203) and (210)
2+	20 30	0.1227*	(005)
1+	21 36	0.1355**	(204)
1	32 52	0.2945	(403) and (320)
1	34 16	0.3170	(225)
1	34 49	0.3260	(410), (306) and (008)
1	38 44	0.3915	(307)

TABLE III.
Diffractionlines of β-Beryllium.
(Iron-radiation).

Int.:	Θ:	$sin^2\,\Theta$ (observed):	$sin^2\,\Theta$ (for copper-α-radiation) observed:	Indices $\{h\,k\,l\}$ calculated from the formula:
2	16° 6½	0.0769	0.0488	(110) and (003)
3	16 56	0.0848	0.0538	(111)
2	18 5½	0.0982**	0.0622	(200)
1*	20 23½	0.1214	0.0775	(113)
1	21 46	0.1375	0.0848	(202)
2	22 55	0.1516	0.0940	(113)
3	24 23	0.1704	0.1087	(203) and (210)
3	25 5	0.1797	0.1135	(211)
4	26 15	0.1945**	0.1231	(005)
4	27 6	0.2075	0.1311	(212)
3	27 50	0.2179**	0.1355	(204)
1	32 3	0.2816	0.1781	(115)
3	39 6	0.3978	0.2518	(313) and (400)

The calculation of the indices of lines with greater values of $sin^2\theta$

becomes rather arbitrary, as many possibilities occur; the relative intensities were visually estimated, that of (101) of the α-form being taken $= 10$ also in this case.

The indices were, — for $a_0 = 7{,}1$ Å and $c_0 = 10{,}8$ Å; — calculated from the corresponding quadratic formula for *copper*-radiation:

$$sin^2\theta = 0{,}0157 \,.\, (h^2 + hk + k^2) + 0{,}00505 \,.\, l^2$$

and from the analogous equation for the β-radiation.

The majority of the lines observed in these powderspectrograms is also met with in the rotation-spectrograms, as may be seen, if the indices of the latter be transformed into those corresponding to the lattice with $a_0 = 7{,}1$ Å and $c_0 = 10{,}8$ Å. The following examples are a proof of this fact, although a number of spots on the rotationspectrograms do not equally occur in the powderspectrograms:

Spots on rotation-spectrograms:	Corresponding lines on powder-spectrograms:	Spots on rotation-spectrograms:	Corresponding lines on powder-spectrograms:
A. round [0001]:		*B.* round the *b*-axis:	
(200)	(110)	(111)	(111)
(110)	(110)	(200)	(110)
(420)	(410)	(201)	(111)
(201)	(111)	(202)	(112)
(111)	(111)	(203)	(113)
(202)	(112)	(112)	(112)
(112)	(112)	(003)	(003)
(203)	(113)	(005)	(005)
(113)	(113)	(113)	(113)
(205)	(115)		
(115)	(115)		

§ 7. There can hardly be any doubt, therefore, that the β-modification of *beryllium* has also hexagonal symmetry, with $a_0 = 7{,}1$ Å and $c_0 = 10{,}8$ Å, — its axial ratio thus being: $a : c = 1 : 1{,}5211$, which is a little smaller, therefore, than that of the α-form. The β-form is probably monotropic; at temperatures in the vicinity of 650° C. it is produced with an observable velocity, but the content of this β-form in the metallic phase is certainly not greater than about 10%, — as may be deduced from the relative intensities of the diffraction-lines. Once produced, the β-form, although metastable, remains present in the metal even at ordinary temperatures, — its transformation into the α-modification obviously being impeded by strong passive resistances.

Beryllium-powder, which yielded strong abnormal spectrograms, was

repeatedly and during a long time, cooled in a mixture of solid carbon-dioxide and acetone, and every time a powderspectrogram of the cooled metal was prepared. In this way it could be stated that the intensities of the extra-lines gradually diminished, although they never completely disappeared. Perhaps this fact can serve as a proof, that at least a "partial" transformation of the β-modification into the stabler α-form is possible. By previous measurements [1]) with heated *beryllium*, we gathered the impression, that the specific weight does *not* considerably alter in this transformation; probably it is a little increased (1,91). If this be true, then the number of the atoms within the elementary cell must be about 60. As the β-modification, however, cannot be prepared in a pure state and its quantity in the metallic phase always remains very limited, more certain data about this question could, at this moment, not be obtained.

The ordinary metal seems always to contain a certain percentage of the metastable modification; this fact, most probably, accounts for the irregular and inconstant physical properties of *beryllium*, as observed by different investigators. [2])

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[1]) F. M. JAEGER and E. ROSENBOHM, loco cit., p. 1060.

[2]) Conf. E. J. LEWIS, loco cit.

Plantkunde. — *Temperatuur en Tijd van den Bloemaanleg bij Bol-Irissen.* (Mit Zusammenfassung). Door A. H. BLAAUW. (Mededeeling N⁰. 39 van het Laboratorium voor Plantenphysiologisch Onderzoek te Wageningen).

(Communicated at the meeting of June 24, 1933).

In de levensgeschiedenis van een volledige knop hebben wij de bladvormende, de bloemvormende en de strekkingsperiode onderscheiden. Wanneer het ooit mogelijk zal zijn over de periodiciteit in de ontwikkeling der gewassen een beter begrip te krijgen in verband met de heerschende groeifactoren, dan dient in de eerste plaats die periodieke vorming en strekking van blad en bloem bij verschillende uiteenloopende gewassen goed bekend en *beschreven* te zijn. Daarna volgt een *vergelijking* en zoo mogelijk een *rangschikking* in verschillende typen van ontwikkeling. Dan pas is er een grondslag om te zoeken, òf er eenig begrijpelijk *verband* is te vinden tusschen bepaalde periodiciteitstypen en de omstandigheden, waaronder die gewassen het jaar doorbrengen; zoowel in de cultuur onder vaak sterk gewijzigde omstandigheden als in het land, waar zij wild voorkomen en waarvan zij het klimaat dus sinds onheuglijke tijden als hun natuurlijk milieu

hebben te aanvaarden. Bij dit zoeken naar een redelijk verband tusschen de periodieke ontwikkelingswijze en de omringende groeifactoren kan verder de *experimenteele ontwikkelingsleer* groote diensten bewijzen. Want zij beperkt zich niet tot het bestaande milieu en tast niet enkel rechtstreeks naar de gunstigste levensomstandigheden, maar zoekt ook vooral naar de *grenzen* waarbinnen de ontwikkelingsprocessen zooals bladvorming, bloemvorming en strekking mogelijk zijn. Juist de kennis daarvan zal ons eenmaal beter doen begrijpen welke mogelijkheden en uiterste grenzen door het klimaat in de wereld van thans en van vroeger tijden aan de verschillende plantensoorten gesteld worden.

Voorloopig beschikken wij over te weinig gegevens om algemeene conclusies te kunnen opsporen. Meer experiment en betere kennis van de jaarlijksche ontwikkeling bij meer soorten van gewassen zijn daarvoor noodig. Een dergelijk vergelijkend en experimenteel onderzoek, dat voor de plantkundige wetenschap op den duur en in menig opzicht direct voor de toepassing in de cultuur in verschillende opzichten van belang kan zijn, vergt veel tijd en geduld, — en het is zelfs zeer waarschijnlijk dat conclusies van meer algemeenen aard en ruimer beteekenis pas in veel later tijden mogelijk zullen zijn voor een volgend geslacht, als omtrent de chemisch-physiologische gebeurtenissen, die de wording van organen en vorm beheerschen, eerst meer kennis en inzicht is verkregen. Het bovenstaande moge een verontschuldiging zijn voor de betrekkelijk oppervlakkige gegevens, die wij in deze en andere meedeelingen brengen, maar tevens een toelichting, waarom wij meenen dat een uitgebreide kennis dezer gegevens — vooral voor de verdere ontplooiïng der plantkunde in de toekomst — noodzakelijk is.

Over de periodieke ontwikkeling van Bol-Irissen is een onderzoek in ons laboratorium in gang. Daaruit willen wij hier een korte mededeeling doen betreffende den tijd van de bloemvorming. Onderzocht werd voorloopig de zeer veel gekweekte Bol-Iris *Imperator*. Deze behoort tot een groep hybriden die als *Iris filifolia*-variëteiten bekend staan, maar in werkelijkheid niet door kruisingen met de echte *Iris filifolia*, maar met *I. Xiphium praecox* gewonnen zijn. Deze groep is het naast verwant en kan tot één vereenigd worden met de groep der Hollandsche Irissen, hybriden van *I. Xiphium praecox* met *I. lusitanica*, *I. Boissieri* e.a.

De eigenlijke Hollandsche Irissen werden gewonnen door de firma C. G. van Tubergen, de andere groep, waartoe o.a. Imperator en Wedgewood behooren, door de Gebrs. De Graaff te Noordwijk. (Zie J. F. Ch. Dix; C. G. van Tubergen; H. R. Wehrhahn; W. R. Dykes).

Van deze Bol-Iris sterven de bladen tegen het einde van Juli af, de bollen worden meest begin Augustus gerooid. De kleine *ronde* bollen boven 5 tot 6 c.M. omtrek zullen het volgend jaar algemeen bloeien, maar na dien bloei oogst men enkele bolletjes, die als okselknoppen in den moederbol zijn ontstaan. De meesten hiervan kunnen pas over 2 of 3 jaar bloeien. Wij komen daarop later terug. Het komt er voor den kweeker op aan de bollen

bij een bepaalde grootte te verkoopen in den zomer vóór het jaar van den bloei. Wij kozen dus voor ons onderzoek bollen van 7 tot 9 cm. omtrek, die zeker het volgend jaar bloeien. Zij werden 29 Juli '32 door ons ontvangen, bij 23° C. bewaard tot 20 Sept., daarna in 17° C. tot 19 October en vervolgens weer geplant.

Wanneer wordt in deze bollen de bloem aangelegd? Bij het rooien bestaan de bolletjes van 7—9 cm., die een uiterst gelijkmatige samenstelling vertoonen, uit gemiddeld *10* nog levende phyllomen, en wel 3 dikke rokken, 1 nog vrij dik blad en verder gemiddeld 6 blaadjes (varieerend van 5 tot 7). Den 19en Oct. is dit aantal van 10 (9,97 uit 15 ex.) op 11½ (11,47) gestegen. In ruim 2½ maand is er slechts 1½ loofblad gevormd door het eindgroeipunt van den bol (afgezien dus van vormingen aan de okselknopjes). Uit dezen uiterst langzamen aanleg van de organen bij 23°—17° bleek reeds, dat wij bij dit bolgewas met een geheel ander geval te doen hadden dan met de ons tot dusver bekende voorbeelden van Hyacinth, Tulp e.a. Wel konden wij experimenteel vaststellen, dat bij lagere temperaturen het tempo der loofbladvorming iets sneller gaat, zooals wij in een andere mededeeling zullen beschrijven. Maar in October bij het planten is er nog geen sprake van bloemvorming.

Daarna werd elke 2 of 3 weken weer een 15-tal gerooid en gefixeerd. Zal de bloemvorming nog vóór den vollen winter in den grond plaats vinden of pas in het voorjaar? Wij kunnen hier volstaan met de fixatie van 28 Dec. te vermelden. Het totale aantal bladdeelen bedroeg nu gemiddeld 13 (13,10), zoodat er sinds 19 Oct. weer 1½ loofblad bijgevormd is in 10 weken bij een temperatuur op 10 cm. diepte in den bodem.

Hoewel wij nog volkomen in de bladvormende periode zijn, valt er een typische verandering aan het groeipunt waar te nemen. De blaadjes worden bij de Iris, gelijk bekend is, in één vlak, afwisselend aan de eene en de andere zijde van het groeipunt afgesplitst. Meet men nu de breedte van het groeipunt, dus loodrecht op het vlak der bladen, dan bedraagt deze gemiddeld 210 μ op 19 Oct. (187 tot 240 μ), maar 28 Dec. gemiddeld 290 μ (213 tot 347 μ). Tijdens den aanleg van 1½ blad is dus tevens het groeipunt vrij aanzienlijk vergroot.

De temperatuur op 10 cm. diepte werd sinds de planting 3 × daags waargenomen, n.l. om ± 7 u. (na 1 April ± 6 u.), om ± 2 u. en 's avonds ± 11 uur. Om een indruk te geven van de gemiddelde temperatuur, waarbij de bollen in deze maanden verkeeren, volgt hier de gemiddelde temperatuur in decaden:

22—31 Oct.	9°,1 C.	11—20 Dec.	2°,2 C.	1—10 Febr.	3°,9 C.	21—31 Mrt.	6°,6 C.
1—10 Nov.	6°,9 „	21—31 Dec.	3°,0 „	11—20 Febr.	1°,4 „	1—10 Apr.	9°,2 „
11—20 Nov.	3°,8 „	1—10 Jan.	4°,3 „	21—28 Febr.	−0°,1 „	11—20 Apr.	9°,5 „
21—30 Nov.	4°,6 „	11—20 Jan.	0°,9 „	1—10 Mrt.	5°,2 „	21—30 Apr.	8°,9 „
1—10 Dec.	2°,3 „	21—31 Jan.	−1°,2 „	11—20 Mrt.	6°,5 „		

Na 28 Dec. treden vorstperioden op (in den grond) van 16 Jan.—4 Febr. en van 20—28 Febr. Fixaties werden uitgesteld tot 1 Maart.

Op 1 Maart is sinds 28 Dec. het aantal bladdeelen van 13.10 tot 14.07 gestegen (waarvan 7 of 8 loofblaadjes zijn); er is in deze 2 maanden ondanks zeer lage temperaturen nog een blad bijgevormd. Maar bovendien blijkt de Iris juist *op de grens van de bladvormende en de bloemvormende periode te staan:*

Van 15 exemplaren verkeeren 4 nog volkomen in de bladvormende periode (stadium I), het groeipunt is 280 tot 290 μ breed. Een 6-tal verkeert in stad. II, 't laatste blaadje is afgesplitst, de breedte van het groeipunt is daarbij thans 400 tot 425 μ. (Fig. 1 op 1 Maart). Dit is het primordium voor de hoofdbloem en de zijbloem. Bij 2 bollen wordt het primordium voor de zijbloem in de oksel van het bovenste blad afgesplitst van het hoofdgroeipunt; bij 3 bollen is dit zijgroeipunt reeds zelfstandig; het primordium van de eindbloem is dan afgerond driehoekig, in 2 gevallen met een zwakke aanduiding van indeeling in 3 primordia.

De volgende fixaties hadden nu plaats op 17, 24, 30 Maart en 7 April.

Op 17 Maart is bij alle bollen de eindbloem volop in wording; twee bloemen bestaan nog pas uit 3 ongedifferentieerde primordia, bij de andere 13 bloemen zijn deze primordia gedifferentieerd in de 3 buitenste tepalen en de 3 (= buitenste) meeldraad-primordia, terwijl de primordia der 3 binnenste tepalen bij de meesten als zwakke kleine bobbels zichtbaar zijn. (Fig. 1). De geheele bloem is nu 670 tot 800 μ breed. Het zijgroeipunt splitst een blad af, dat geadosseerd staat ten opzichte van de eindbloem, terwijl de rest primordium is voor de zijbloem. Op 24 Maart zijn de primordia der binnenste tepalen bij nagenoeg alle bollen geheel zelfstandig; er is nog

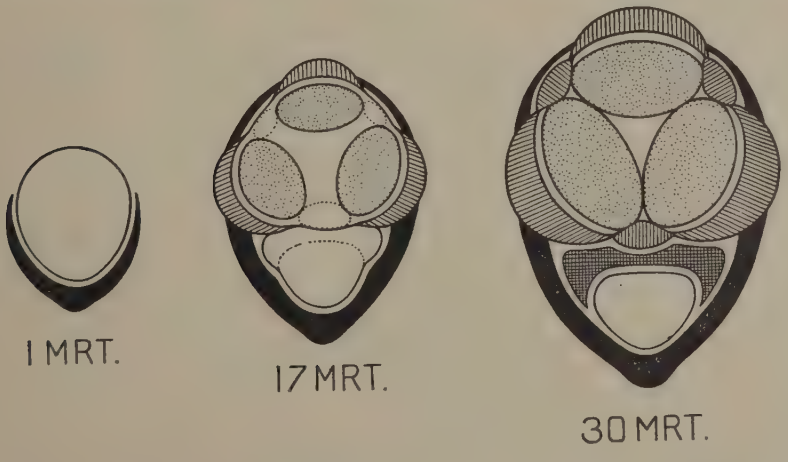

Fig. 1. Schema van den gemiddelden toestand en omvang van den bloemaanleg bij de Bol-Iris „Imperator" in Mrt. 1933. Op 1 Mrt: primordium voor hoofd- en zijbloem met het bovenste loof-schutblad; 17 Mrt. en 30 Mrt.: bloemdekbladen gearceerd, meeldraden gestippeld. (Vergr. 40×).

geen begin van vruchtbladen. Op 30 Maart wordt in 2 v. d. 15 bollen een eerste begin van den vruchtblad-aanleg waargenomen. De bloem is gemid-

deld 900 μ in doorsnee (Fig. 1). Op 7 April zijn in 11 van de 13 bloemen de vruchtblad-primordia aan den voet der meeldraden te vinden (in 2 bollen was geen bloem gevormd).

Alle bloemkransen zijn dus in de 1e helft van April primordiaal aanwezig. Van 24 Maart tot 7 April is de hoogte van de bloem $2\frac{1}{2} \times$ grooter geworden (van gemidd. 0,6 mm. tot 1,6 mm.). Daarbij zijn de 3 meeldraden overheerschend in omvang. In de nu volgende weken moet er nog een geheele wijziging komen in de onderlinge proporties der organen, vooral wat betreft de nu nog kleine primordia der vruchtbladen aan den voet der meeldraden, die straks met hun groote bloembladachtige stijlen geheel over den meeldraad zich zullen uitbreiden.

Wij zullen daarin hier niet verder treden ; de bloemaanleg en -vorming wordt later nader beschreven in de verhandeling over de periodieke ontwikkeling van de „Hollandsche" Bol-Iris.

Het is de bedoeling hier alleen de aandacht te vestigen op den tijd, waarin de Bol-Iris de primordia der bloemkransen gevormd heeft.

Daar de bloemvorming bij een zoo opvallend lage temperatuur plaats had, werd, behalve het door ons zelf behandelde materiaal, ook nog een partij door een kweeker behandeld en bij een bloemist in November buiten geplant, door ons nagekeken. Op 29 Dec. '32 was ook hier het groeipunt nog in de bladvormende periode, zonder bloemaanleg. Op 14 Maart was de bloem nagenoeg in hetzelfde stadium als bij ons op 17 Maart, slechts een weinig verder, wat de ontwikkeling der binnenste tepalen betreft ; de bloemaanleg was iets forscher, doordat het dikkere bollen waren dan de onze.

Vervolgens hebben wij nog een Spaansche Iris op 28 Maart onderzocht. Deze groep der Spaansche Irissen, waarvan de stamvorm *Iris Xiphium* is, bloeit ongeveer 14 dagen later dan de Hollandsche Irissen. 't Was dus de vraag of ook hier in Maart reeds een bloemaanleg aanwezig zou zijn. De 15 bollen hadden allen bloemaanleg, 7 nog zonder vruchtblad-primordia, 6 met een rand onder langs de meeldraden als eerste begin der vruchtbladen en 2 met sterk ontwikkelde vruchtblad-primordia. Deze Iris is dus einde Maart zeker even ver ontwikkeld als Imperator.

Vergelijken wij nu deze Bol-Irissen met de wortelstok-Iris, zooals *I. germanica,* dan blijkt bij deze de periodieke ontwikkeling geheel anders te zijn. Men vindt hierover een aanwijzing bij S. KRÄMER (1932), Physiologische Studien an Iris Germanica, S. 247 : „Bei gut entwickelten Trieben wurden die ersten Blütenanlagen Mitte Juni unter dem Mikroskop beobachtet."

Zooals bij de meeste tot dusver door ons onderzochte gewassen ontstaat de bloem bij de wortelstok-Iris in den voorgaanden zomer, om pas 11 maanden later (half Mei à 1 Juni) te bloeien. De hier onderzochte Bol-Irissen leggen de bloem in Maart en begin April aan en bloeien dan kort daarop in de 1e helft van Juni. Wat ons daarbij nu het meeste treft is wel de lage temperatuur, waarin deze Bol-Irissen hun bloemen moeten vormen.

Zij raken midden in den winter gereed met den aanleg der loofbladen en staan zoo in den loop van Februari voor den bloemaanleg. Daar volgt ook reeds uit, dat die bladvorming bij bijzonder lage temperatuur nog voortschrijdt, zoodat van 28 Dec. tot 1 Maart nog een loofblad en bij verscheiden bollen reeds de eerste inleiding van de bloemvorming tot stand komt, terwijl er in 1933 in Jan.-—Febr. toch vrij veel vorst was. Zie de gemiddelde temperatuur per decade in bovenstaand lijstje ; het daggemiddelde bedroeg in deze twee maanden slechts 1 × ruim 7° C., 2 × ruim 6°, 4 × ruim 4° en bleef op 51 dagen beneden 4°, waarvan 38 dagen beneden 2° C.

Tot dusver hadden wij uit proeven en periodiciteitsonderzoek ervaren, dat terwijl de strekking nog langzaam voortgaat, de bladaanleg, dus de celdeeling, in den regel enkele graden boven het vriespunt practisch stil staat. Zoo vindt men, dat de jonge knop van de Tulp in Nov. ophoudt met bladafsplitsing, om pas met April weer voort te gaan (R. MULDER en I. LUYTEN 1928) ; ook bij de Narcis (E. HUISMAN en A. M. HARTSEMA 1913, Tab. 4) staat de bladvorming na half November stil tot in Februari.

Zoo leek het wel, alsof de winterkou van zelf reguleerend werkte op de periodieke ontwikkeling, in zooverre, dat door dezen stilstand in landen met vrij strenge winters de belangrijke bloemvorming vanzelf nooit in den winter zou plaats vinden en dus niet door strenge vorst kon overvallen worden. Wij weten nu, dat de Bol-Iris in den winter met blad- en bloemvorming voortgaat, dat de bloemaanleg in 1933 onmiddellijk na de Februari-vorst plaats vindt bij een gemiddelde dagelijksche temperatuur in Maart van 4° tot 8° C. (5.2—6.5— en 6.6 per decade) op 10 cm. in den grond. Bij die temperatuur worden uitstekende bloemen gevormd, welke in de eerste helft van Juni '33 hebben gebloeid.

Vergelijken wij daarmee nu de Tulp, die toch tegen vrij lage temperaturen bestand is en waarvan het temperatuur-karakter nog ± 5° lager ligt dan bij de Hyacinth. Wanneer de Tulp in Juli de bladvorming heeft afgemaakt heeft het experimenteele onderzoek van de ontwikkeling ons geleerd, (I. LUYTEN, G. JOUSTRA en A. H. BLAAUW, 1925, Fig. 3), dat in 5° C. de bloemvorming wel op gang komt, maar veel langzamer dan bij de Bol-Iris in Maart bij ± 6° C.

Uit een nog niet gepubliceerde proef kunnen wij vermelden dat in 5° de Darwintulp *W. Copland* pas na ± 3½ maand vruchtblad-primordia heeft, *Pride of Haarlem* komt daaraan in 5° (Aug. tot Maart) in 't geheel niet toe. De Tulp brengt na een zomerbehandeling in 5° C. geen normale bloemen tot ontwikkeling.

In Fig. 2 wordt de grond-temperatuur van 20 Febr.—30 April 1933 door een zwarten band aangegeven, die door de dagelijksche maxima en minima begrensd wordt; binnen die temperaturen had de bloemaanleg in Maart en begin April bij de Iris plaats. Ter vergelijking is nu in de bovenhelft van de figuur (gestippelde band) de grondtemperatuur in Mei en Juni 1931 afgezet, omdat we weten, dat o.a. bij de Narcis in *die* weken de

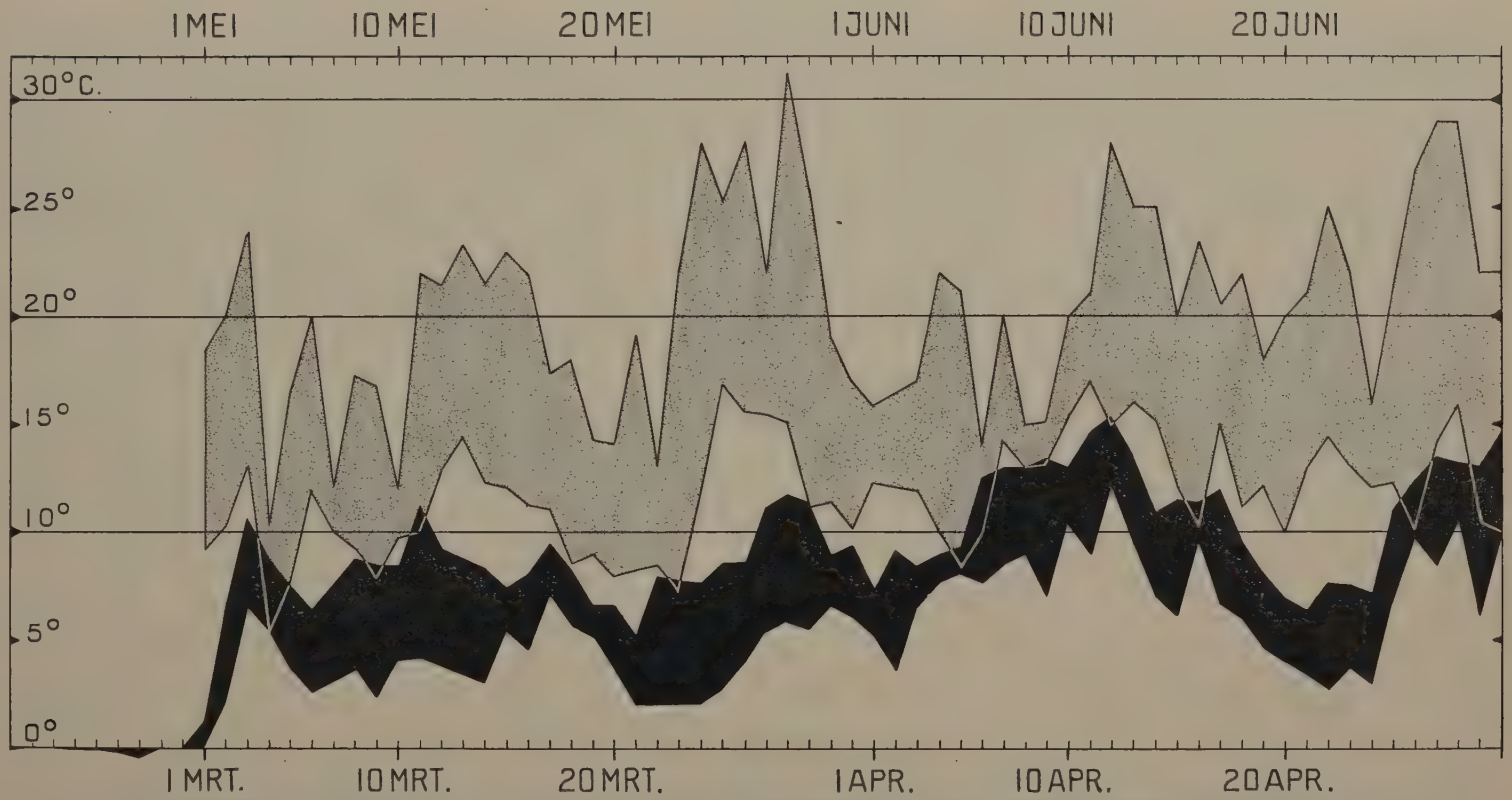

Fig. 2. Dagelijksche temperatuur op 10 cm. in den grond (maxima en minima tot een band verbonden). Een zwarte band van 20 Febr.—30 April '33: binnen deze temperaturen vindt de bloemaanleg van deze Iris plaats; een gestippelde band van 1 Mei—1 Juli '31, waarbinnen bijv. de Narcis bloem vormt.

bloemvorming plaats heeft in den grond. Men ziet onder welke verschillende conditie de bloemaanleg bij Bol-Iris en bij Narcis e.a. verloopt. Bij de Bol-Iris is midden Maart, bij de Narcis 1 Juni de aanleg in vollen gang; de gemiddelde grondtemperatuur van Maart '33 was 6°.14 C., van 16 Mei —16 Juni '31: 15°.65 C. Daarbij is vooral opvallend, dat de aanleg der primordium-kransen in die lage temperatuur bij de Bol-Iris volstrekt niet bijzonder langzaam verloopt: Van het ongedifferentieerde bloemprimordium af (gemiddeld 1 Maart) totdat de kransen T I, T II en M I tot stand zijn gebracht (tegen 24 Maart) heeft de Iris in nauwelijks 6° C. ruim 3 weken noodig; totdat ook de vruchtblad-primordia aangelegd zijn, verloopen in 't geheel ruim 5 weken (waarbij de temperatuur van 26 Maart tot 7 April vrij geleidelijk van 7° tot 10° C. stijgt). Dat de krans der vruchtbladen iets langer op zich laat wachten na den aanleg der vorige kransen is een gewoon verschijnsel, bijv. ook bij Tulp en Narcis in den zomer.

Belangrijk is het, dat deze Bol-Iris ons zoodoende voor ervaringen plaatst, die weer sterk afwijken van hetgeen wij tot dusver van de periodieke ontwikkeling van andere planten waren tegengekomen. Dit zal een aansporing zijn het vergelijkend onderzoek voort te zetten en na te gaan welke andere gewassen het periodiciteits-type van deze Irissen volgen. Ook zullen wij den tijd en de temperatuur van den bloemaanleg bij andere soorten Bol-Irissen onderzoeken, — en ten slotte ook nagaan of deze Imperator werkelijk ieder jaar in dezelfde maand de bloem aanlegt, of dat zij hiermee ook wel vroeger gereed komt, hetgeen men voor zachte wintermaanden wel zou verwachten, maar wat dan wederom de vraag stelt of invallende vorst dan door den jongen aanleg verdragen wordt.

Wageningen, Juni 1933.

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ZUSAMMENFASSUNG.

Die Temperatur und die Zeit der Blütenanlage bei der holländischen Zwiebel-Iris.

Die holländischen Zwiebel-Irissen bilden eine Hybriden-Gruppe, welche hauptsächlich bei zwei niederländischen Züchtern durch Kreuzung der süd-

spanischen *Iris Xiphium praecox* mit mehreren anderen süd-europäischen und nord-afrikanischen Iris-Arten entstanden ist. Eine der für den Handel wichtigsten Hybriden „Imperator" haben wir für eine entwicklungsgeschichtliche Untersuchung der Zwiebel-Iris gewählt. Aus dieser Arbeit wird hier eine vorläufige Mitteilung gegeben.

Die kleinen runden Zwiebeln welche im Sommer einen Umfang von wenigstens 5—6 cm. erreicht haben, blühen im nächsten Frühsommer (Ende Mai, Anfang Juni). Zwiebeln von 7—9 cm., welche für die Untersuchung gebraucht wurden, bestehen 29 Juli aus ± 10 noch lebenden Phyllomen: 3 dicken Röcken, 1 halb-dicken Rock und weiter durchschnittlich 6 Blättchen. Vom 29 Juli bis 20 Sept. in 23° C und bis 19 Okt. in 17° C aufbewahrt, zeigen die Zwiebeln 19 Okt. 11½ Phyllome. In fast 3 Monaten sind nur 1 bis 2 Blättchen gebildet. Dies zeigt uns schon, dass der Entwicklungsprozess sehr bedeutend von demjenigen von Narcissus, Hyacinthus, Tulipa u. A. abweicht. Am 19en Okt. werden die Zwiebeln gepflanzt. Ende Dez. sind ± 13 Blätter vorhanden, am 1en März 14, davon 7 oder 8 Laubblätter. Der Winter war kalt mit zwei Frostperioden, wovon die letzte am 1en März endete (siehe Temperaturen in Dekaden in der kleinen Tabelle.) Gerade jetzt stehen die Zwiebeln am Anfang der Blütenbildung; nur 4 von 15 fixierten sind noch in der blattformenden Periode; 6 haben das letzte Laubblatt, halb-Bractee, angelegt, der Vegetationspunkt ist von 290 μ bis 415 μ breiter geworden und ist jetzt Primordium für die Hauptblume und eine Seitenblume (Fig. 1 März 1); 5 Zwiebeln zeigen den Anfang der Blütenbildung durch Abtrennung des Nebenvegetationspunktes für die Seitenblume und durch die abgerundet-dreieckige Form des restierenden Primordiums. Am 17en März ist die Blütenbildung schon weit vorgeschritten, die äusseren Tepalen und Staubblätter sind selbständige Primordia, die inneren Tepalen sind im Anfang der Bildung begriffen (Fig. 1 März 17). Das Primordium der Nebenblume ist selbständig, nachdem es ein Blatt geformt hat, das noch nicht ganz abgetrennt ist. Schon am 24en sind die inneren Tepalen fertige Primordia; am 30en März hat die Anlage sich vergrössert (Fig. 1 März 30); in 2 von 15 Fällen sind die Fruchtblatt-primordia da; — am 7en April fast alle. Darauf findet das Auswachsen der Formen statt.

Wichtig ist nun die ausserordentlich niedere Temperatur wobei diese Anlage stattfindet, so wie wir es bis jetzt in unseren Periodizitätsuntersuchungen noch nicht begegneten. In Fig. 2 findet man Maxima und Minima zu einem Band vereinigt: schwarz vom 20en Februar bis 1. Mai; in März—April findet zwischen diesen Temperaturen die Blütenbilding bei dieser Iris statt; darüber ein punktiertes Band vom 1. Mai bis 1. Juli, zwischen welchen Temperaturen die Anlage z.B. bei *Narcissus* stattfindet. Bei dieser Zwiebel-Iris also in März bei 6°.14 C, im Durchschnitt bei *Narcissus* bei 15°.65 C. Für diese niedere Temperatur geht die Blütenbildung in 6° nicht einmal so langsam. Vergleichen wir hiermit unsere Experimente bei der Tulpe, derer Temperaturcharakter

schon weit niedriger liegt als bei *Hyacinthus*. In 5° C geht die Blütenbildung bei *Tulipa* sehr langsam vor sich, hat erst nach 3½ Monat die Karpellen bei der einen Varietät angelegt, bei einer anderen kommt es überhaupt nicht mehr zur Karpellen-bildung. Richtige Blumen entfalten sich nach 5° C (in Juli—Okt.) nicht. Bei der Zwiebel-Iris sind die Karpellen in 6° C nach 5 Wochen als Primordien sichtbar, und es entfalten sich grosse kräftige Blumen Ende Mai.

Auch die sogenannten spanischen Irissen, welche 14 Tage später blühen haben die Anlagen gleichfalls Ende März fast fertig. Wir werden näher untersuchen wie andere Zwiebel-Irissen sich verhalten und wie auch dieselbe Iris in anderen Wintern; ob es vorkommt, dass die Blüte schon früher gebildet wird, wie man es für milde Winter und für gewisse andere Arten wohl annehmen muss, — was dann aber die Frage aufwirft, ob diese Anlagen einen plötzlich eintretenden Frost ertragen können.

Diese Art Iris ist für uns von Bedeutung weil sie uns einen anderen Periodizitätstypus zeigt, als die bis jetzt von uns beschriebenen Gewächse. Erstens weil sie die Blattbildung fertig macht in der sehr niederen Wintertemperatur: bei *Narcissus* und *Tulipa* steht von Nov. bis Febr. und April die Blattbildung still und somit kommt die blütenbildende Periode resp. erst in Mai und Juli; zweitens weil die Iris die Blütenbildung, welche durch die ablaufende blattformende Periode schon in März anfangen soll, auch wirklich bei so niederer Temperatur ganz normal zu Stande bringen kann; drittens, weil sie nach der Anlage sofort zur Streckung und zum Blühen übergeht, während die meisten mehrjährigen und darunter alle bis jetzt von uns beschriebenen Gewächse im vorigen Sommer ihre Blüten anlegen. Im selben Jahre anlegen und blühen findet man nicht nur bei den einjährigen Pflanzen aber auch bei gewissen mehrjährigen z. B. bei Gladiolen und Herbst-Krokus.

Die Untersuchungen der periodischen Entwicklung müssen noch an mehreren Gewächsen verfolgt werden um einen Vergleich verschiedener Typen durchführen zu können.

Physiology. — *Musculo-muscular "story" ("étage") reflexes of dogs whose spinal chord has been cut in several places.* By G. VAN RIJNBERK and J. TEN CATE.

(Communicated at the meeting of June 24, 1933).

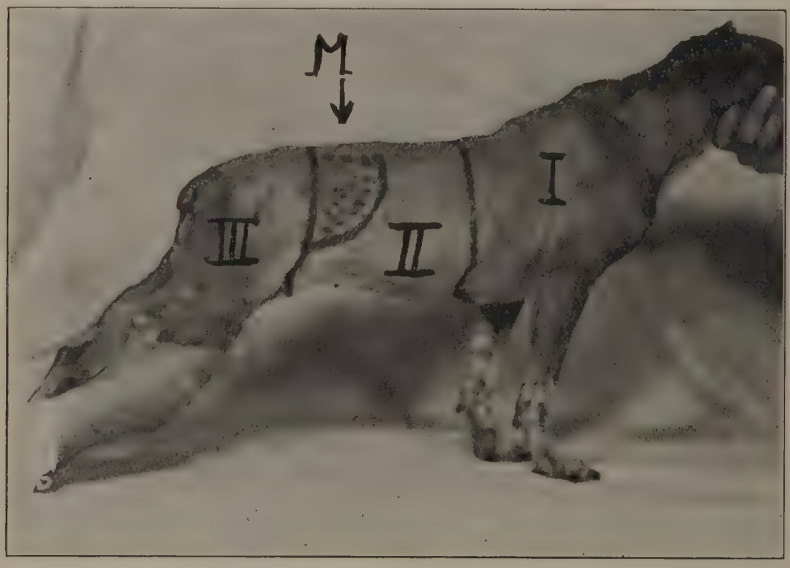
Upon an examination of dogs of whose spinal chord months beforehand a broad section had been put out of action and completely separated by two cross-cuts from the remaining central nervous system, we often found, months after the operation, a phenomenon which we want to describe here.

For a better understanding the following may be previously observed.

a. The spinal chord of our animals was, in consequence of the operation, divided into three different sections: a front (cranial) section to about Th. 8, connected with the cerebrum; a middle section from about Th. 9 to Th. 13 (L. I), totally deprived of its dorsal and ventral roots and separated by two cross-cuts from the remainder of the central nervous system. and finally a hindmost (caudal) division, beginning at L. I, not connected with the cerebrum, but by means of its roots in afferent and efferent connection with skin and muscles.

b. The skin of those animals can, in consequence of the operation on the spinal chord, functionally be distinguished into three different regions. The skin of the front part of the trunk, the fore-legs and the head was sensitive and reflex; a zone of skin round the middle section of the trunk was for about 6 dermatomes insensitive and areflex; the skin of the hind-part of the trunk was insensitive but reflex.

c. The muscles of the animals could likewise be divided into three territories: a front section, to be put in action by the will and reflexly;



Dog S. I. On 18 Nov. 1932 the spinal chord is cut between Th. 7 and 8 and between Th. 13 and L. I. All ventral and dorsal roots of the spinal chord from Th. 8 to Th. 13 on the right are cut as well. I. Normal front part of the animal. III. Reflex hind-part. II. Sensorily, reflexly and motorily paralyzed middle section. M. Receptive field (tapping), apparently situated in the skin of this last section, for the musculo-muscular "story"-reflexes in myotome L. I and II. Photo taken on 3 Febr. 1933. The actual reflexogenic zone appeared to consist by the end of May of a section of muscles of the Mm. longissimus dorsi and ilio costalis innervated by L. I and L. II.

a middle section, neither by will nor reflexly attainable, so completely paralyzed and, owing to the destruction of the spinal chord, at the same time fairly atrophic, and finally a caudal section beginning at the first lumbar myotome, not attainable by cerebral impulses, yet reflexly to be brought to contraction from the caudal region of the skin.

It now appeared repeatedly that tapping with the finger or knocking with a percussor in a certain region of the skin, within the insensitive areflex zone, close to the dorsal midline (v. picture), led to clearly perceptible contractions of the muscles in parts of the long dorsal muscles and in muscles of the abdominal wall. These regions of the muscles lay behind (caudally of) the totally paralyzed zone of muscles.

In order to ascertain where the point of application of the mechanical stimulus was situated — for the skin which was not connected with the functioning spinal chord could not be taken into account here! — we then proceeded as follows.

1. The skin of the apparently reflexogenic region was removed. Tapping on the subcutaneous fat-tissue underneath appeared to be able undisturbedly to wake the reflex contraction.

2. The fat-tissue was removed. By tapping on the fascia of the long dorsal muscles or scratching with the needle we could woke the reflex undisturbedly.

3. The fascia was detached. Pulling at the fascia uworked the reflex. Likewise scratching or tapping on the long dorsal muscles or bringing them to contraction by means of a faradic stimulus.

From this it is clear that the phenomenon observed by us is a purely musculo-muscular reflex.

By a broad removal of the skin and fat-tissue we have further been able to ascertain that the contracting muscles consisted in the first place of the percussed dorsal muscles themselves and besides of the most caudal parts of the abdominal wall, especially the most caudal part of the Obl. ext. and of the Rectus. At the same time it appeared that stimulation of these muscles also occasioned musculo-muscular contraction in the dorsal muscles (longissmius dorsi and iliocostalis). Consequently the reflex action is mutual from dorsal to ventral muscles and reversely.

The sections of the muscles stimulated in these experiments, as well as those which exhibited the reflex contraction, all belong to the same two myotomes: L. I and L. II from the caudal reflex region of the body. The reflex described here is consequently a uni- or bisegmental "story" or "étage" reflex. We did not succeed in causing contractions in the just mentioned sections of the muscles by stimulation of more caudally situated muscles.

In the hind-legs tapping on the bones (femur, tibia) and on the separate muscles caused reflex contraction in numerous muscles, occasioned by the proprioceptive stimulus. Probably here also the reaction remains restricted

within a circumscribed reflex-story of the spinal chord. But we did not accurately examine this question which is not easily answered.

From our experience one may perhaps conclude that these segmental musculo-muscular "story"-reflexes often belong to the Isolierungsveränderungen (MUNK) of the spinal chord. For the physiology of the proprioceptive reflexes of muscles it is important that both, thickening by contraction and lengthening of the muscle fibres by stretching, can stimulate the reflex.

Geodesy. — *Nets of Triangles Consisting of Points with Circular Error-Curves.* By J. M. TIENSTRA. (Communicated by Prof. F. A. VENING—MEINESZ.

(Communicated at the meeting of June 24, 1933).

1. *Introduction.*

When by measurement the position of a point in a plane has been determined, the accuracy of this determination is represented by the error-curve of the point according to the theory of the least squares. This curve is a foot-point curve of an ellipse. The determined point is the centre of this curve.

In particular cases the error-curve is a circle. The point is then determined with the same accuracy in all directions.

Generally it will deserve recommendation to require equal accuracy in all directions when determining a point. This will e. g. be the case with an angle point of a net of triangles.

In the following exposition the thesis will be proved that:

IF THE ANGLES OF A NET OF TRIANGLES WITHOUT OBTUSE ANGLES, ARE MEASURED WITH WEIGHTS WHICH ARE PER TRIANGLE PROPORTIONAL TO THE COTANGENTS OF THE ANGLES, ALL THE POINTS OF THE NET WILL HAVE CIRCULAR ERROR-CURVES. (The influence on the accuracy of the position of the points as a consequence of errors of observation in the measurement of the base is left out of consideration).

2. *The drawing-up of the normal equations.*

We will prove the thesis for the net of triangles drawn in fig. 1. It is easy to see by the proof that the thesis is of general validity. A measured angle is indicated by α, β, or γ provided with an index indicating the triangle in which the angle lies. Thus α_{12} is angle α in $\triangle$ 12.

The weights with which the angles are measured, are per $\triangle$ proportional to the co-tangents of these angles.

Hence we may write:

Measured angles	Weights
α_i	$g_i \cot \alpha_i$
β_i	$g_i \cot \beta_i$
γ_i	$g_i \cot \gamma_i$

There are in our net 39 measured angles, which must satisfy $13+3+3=19$ relations.

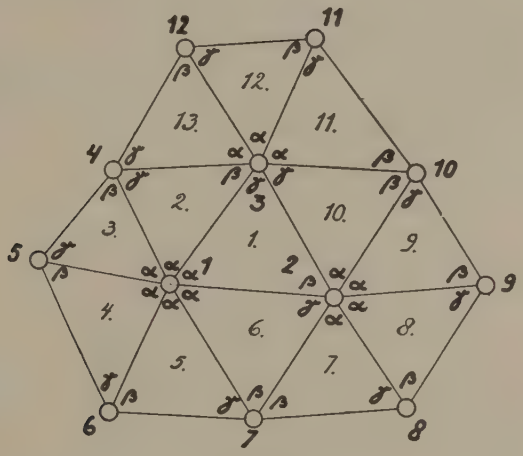


Fig. 1.

Each of these angles yields the condition that the sum of the angles must be 180°; these conditions will be called *triangle-conditions* in what follows.

Let us denote the corrections to the angles α_i, β_i, and γ_i resp. by v_{α_i}, v_{β_i} and v_{γ_i}; we then have 13 conditions of the form:

$$v_{\alpha_i}+v_{\beta_i}+v_{\gamma_i}=\pi-\alpha_i-\beta_i-\gamma_i=w_{d_i} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Further round every central point (the points 1, 2 and 3) the sum of the angles must be 2π. We will call these equations the *horizon-conditions*.

The horizon condition of the central point 1 runs:

$$v_{\alpha_1}+v_{\alpha_2}+v_{\alpha_3}+v_{\alpha_4}+v_{\alpha_5}+v_{\alpha_6}=2\pi-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6=w_{h_1}. \quad (2)$$

The two other points give analogous forms.

Further in the triangles round each central point the product of the sines of the left hand circumference angles must be equal to that of the right hand circumference angles.

This yields the three sine-conditions, the first of which runs:

$$\left.\begin{aligned} v_{\gamma_1}\cot\gamma_1 - v_{\beta_1}\cot\beta_1 + v_{\gamma_6}\cot\gamma_6 - v_{\beta_6}\cot\beta_6 + v_{\gamma_5}\cot\gamma_5 - v_{\beta_5}\cot\beta_5 + \\ + v_{\gamma_4}\cot\gamma_4 + \\ - v_{\beta_4}\cot\beta_4 + v_{\gamma_3}\cot\gamma_3 - v_{\beta_3}\cot\beta_3 + v_{\gamma_2}\cot\gamma_2 - v_{\beta_2}\cot\beta_2 = \\ = -\Sigma \log\sin\gamma_i + \Sigma\log\sin\beta_i = w_{S_1} \end{aligned}\right\} \quad . \quad . \quad (3)$$

From these conditions the normal equations can now be written down in the usual way. We will, however, simplify this somewhat by the application of an artifice. Through a linear transformation of the conditions first other horizon-conditions are drawn up.

For this purpose we diminish the first of these equations by the 6 triangle-conditions of the $\triangle\triangle$ 1 to 6 (inclusive), after having multiplied them in succession by $\cot\beta_1 \,.\, \cot\gamma_1$; $\cot\beta_2 \,.\, \cot\gamma_2$; . . . ; $\cot\beta_6 \,.\, \cot\gamma_6$.

Taking the equation:

$$\cot\beta \,.\, \cot\gamma + \cot\gamma \,.\, \cot\alpha + \cot\alpha \,.\, \cot\beta = 1 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

into account, we may then write for the horizon-condition of the central point 1 thus formed anew:

$$\left.\begin{aligned} \cot\alpha_1(\cot\beta_1 + \cot\gamma_1)\, v_{\alpha_1} - \cot\beta_1\cot\gamma_1\, v_{\beta_1} - \cot\beta_1\cot\gamma_1\, v_{\gamma_1} + \\ + \cot\alpha_2(\cot\beta_2 + \cot\gamma_2)\, v_{\alpha_2} + \\ - \cot\beta_2 \,.\, \cot\gamma_2\, v_{\beta_2} - \cot\beta_2\cot\gamma_2\, v_{\gamma_2} + \ldots . \\ \ldots . + \cot\alpha_6(\cot\beta_6 + \cot\gamma_6)\, v_{\alpha_6} - \cot\beta_6\cot\gamma_6\, v_{\beta_6} - \\ - \cot\beta_6\cot\gamma_6\, v_{\gamma_6} = \\ = w_{h_1} - \cot\beta_1\cot\gamma_1\, w_{d_1} - \cot\beta_2\cot\gamma_2\, w_{d_2} - \ldots . - \\ - \cot\beta_6\cot\gamma_6\, w_{d_6} = w'_{h_1} \end{aligned}\right\} \quad . \quad . \quad (5)$$

We treat the horizon-conditions of the points 2 and 3 in the same way. For the formation of the normal equations we have now the 13 relations of form (1), 3 relations of form (5), and 3 relations of form (3).

The normal equations become:

$$\left.\begin{aligned} \frac{1}{g_1} D_1\ (tg\,\alpha_1 + tg\,\beta_1 + tg\,\gamma_1) &= w_{d_1} \\ \frac{1}{g_2} D_2\ (tg\,\alpha_2 + tg\,\beta_2 + tg\,\gamma_2) &= w_{d_2} \\ \frac{1}{g_{13}} D_{13}\,(tg\,\alpha_{13} + tg\,\beta_{13} + tg\,\gamma_{13}) &= w_{d_{13}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

$$\left.\begin{aligned} p_{11} H_1 + p_{12} H_2 + p_{13} H_3 &= w'_{h_1} \\ p_{12} H_1 + p_{22} H_2 + p_{23} H_3 &= w'_{h_2} \\ p_{13} H_1 + p_{23} H_2 + p_{33} H_3 &= w'_{h_3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\left.\begin{aligned} p_{11}\,S_1 + p_{12}\,S_2 + p_{13}\,S_3 &= w_{s_1} \\ p_{12}\,S_1 + p_{22}\,S_2 + p_{23}\,S_3 &= w_{s_2} \\ p_{13}\,S_1 + p_{23}\,S_2 + p_{33}\,S_3 &= w_{s_3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The correlatives in (6) are represented by the symbols D_i, those in (7) by H_i, and those in (8) by S_i. Further is in (7) and (8):

$$p_{11} = \frac{1}{g_1}(\cot\beta_1 + \cot\gamma_1) + \frac{1}{g_2}(\cot\beta_2 + \cot\gamma_2) + \ldots + \frac{1}{g_6}(\cot\beta_6 + \cot\gamma_6)$$

$$p_{22} = \frac{1}{g_1}(\cot\alpha_1 + \cot\gamma_1) + \frac{1}{g_{10}}(\cot\gamma_{10} + \cot\beta_{10}) + \ldots + \frac{1}{g_6}(\cot\beta_6 + \cot\alpha_6)$$

p_{33} = idem round central point 3.

$$p_{12} = -\frac{\cot\gamma_1}{g_1} - \frac{\cot\beta_6}{g_6}$$

$$p_{13} = -\frac{\cot\beta_1}{g_1} - \frac{\cot\gamma_2}{g_2}$$

$$p_{23} = -\frac{\cot\alpha_1}{g_1} - \frac{\cot\beta_{10}}{g_{10}}.$$

A quantity p_{ik} is zero when the two points to which it refers, are not connected by a side of the net of triangles. If point 4 was also a central point, p_{24} would be zero.

As appears from (6), (7), and (8), the normal equations present this very important particularity that they have fallen apart into 15 detached groups. Each of the equations (6) immediately yields a correlative D, system (7) gives the correlatives H and system (8) the correlatives S.

In comparison with the usual case, in which the weights are equal, and in which the normal equations are constructed so that they are all connected and form one system, from which in our case all the 19 correlatives had then to be solved, the measurement of the angles with the weights, as we have supposed it, means an important simplification. In the systems (7) and (8) the correlatives have besides the same coefficients, so that they can be solved at the same time, e. g. by GAUSS' method of solution, by taking 2 sets of known members in them.

Further the coefficients of the normal equations are so simple of construction that they are very easy to compute.

When the quantities g_i are all taken equal, it comes to this that the proportionality of the weights to the co-tangents is not valid per triangle, but for the whole net.

We may remark here that the method of adjustment obtained above may also be applied in those cases in which the condition concerning

the weights of the angles has not been fulfilled. Then the method of adjustment is not the correct one, but in such a case it may be considered as a method of approximation, which may be applied in less accurate measurements. In this case it does not matter if angles greater than 90° occur in the net. For the condition that the angles of the net must be smaller than 90°, had to be put, because the co-tangent of an obtuse angle is negative, and the measurement of an angle with negative weight is impossible.

The method of approximation can be made more and more "rigorous", by fulfilling the requirement about the weights of the angle measurement more and more. Thus if the net contains a very acute angle, it would be possible to measure it somewhat oftener, without, however, conforming to the rule of the weights for the whole net. In this way the method of approximations might be cleared of its excrescences.

3. *The error-curves of the points.*

It is required for a circular error-curve that:

$$Q_{xx} = Q_{yy} \quad \text{and} \quad Q_{xy} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

We consider:

$$z = x + iy \qquad (i = \sqrt{-1}),$$

and desire to find the weight value of z. We find:

$$Q_{zz} = Q_{xx} + 2iQ_{xy} - Q_{yy}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Hence if (9) is satisfied, $Q_{zz} = 0$. If inversely $Q_{zz} = 0$, it follows from (10), that the equations (9) must exist. Accordingly for the investigation into a circular error-curve may be substituted the investigation of the weight value of the quantity z.

Let us assume that the connecting line 1—2 in fig. 1 is the base b of the net of triangles, which we suppose to be free from error for this investigation, and let 1 be the origin of the rectangular system of axes x, y with positive x-axis along 1—2, then we have e. g. for point 11:

$$z_{11} = z_3 + (z_{11} - z_3)$$

$$z_3 = b \frac{\sin \beta_1}{\sin \gamma_1} \{\cos(-\alpha_1) + i \sin(-\alpha_1)\} = b \frac{\sin \beta_1}{\sin \gamma_1} e^{-i\alpha_1}$$

$$z_{11} - z_3 = b \frac{\sin \alpha_1}{\sin \gamma_1} \cdot \frac{\sin \alpha_{10}}{\sin \beta_{10}} \cdot \frac{\sin \beta_{11}}{\sin \gamma_{11}} \{\cos(\beta_1 - \gamma_{10} - \alpha_{11}) + i \sin(\beta_1 - \gamma_{10} - \alpha_{11})\} =$$

$$= b \frac{\sin \alpha_1}{\sin \gamma_1} e^{i\beta_1} \cdot \frac{\sin \alpha_{10}}{\sin \beta_{10}} e^{-i\gamma_{10}} \cdot \frac{\sin \beta_{11}}{\sin \gamma_{10}} e^{-i\alpha_{11}}$$

(e is the base of the natural system of logarithms).

We introduce the following quantities:

$$\left.\begin{aligned} \frac{\sin \gamma_j}{\sin \beta_j} \cdot e^{i\alpha_j} &= \xi_j \\ \frac{\sin \alpha_j}{\sin \gamma_j} \cdot e^{i\beta_j} &= \eta_j \quad (j = 1, 2 \ldots, 13) \\ \frac{\sin \beta_j}{\sin \alpha_j} \cdot e^{i\gamma_j} &= \zeta_j \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

It is now immediately seen by the derivation given for z_{11} that for the z of any point of the net may be written:

$$z = f(\xi_1 \cdot \xi_2, \ldots \xi_{13}, \eta_1, \eta_2 \ldots \eta_{13}, \zeta_1, \zeta_2, \ldots \zeta_{13}),$$

in which expression as a rule all these quantities do not occur.

When now for every ξ, η or ζ:

$$Q_{\xi\xi} = Q_{\eta\eta} = Q_{\zeta\zeta} = Q_{\xi\eta} = Q_{\xi\zeta} = Q_{\eta\zeta} = 0$$

Q_{zz} will likewise be zero according to the law of propagation of errors. To show this we calculate $Q_{\xi\xi}$ as an example. We have:

$$\log \xi_1 = \log \sin \gamma_1 - \log \sin \beta_1 + i\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Hence the weight value of a quantity $\log \xi$ must be calculated, which quantity is a function of a number of measured quantities α_1, β_1, and γ_1, between which there exist relations (the condition equations).

We now have:

$$\left.\begin{aligned} Q_{\log \xi_1 \xi_1} = \left[\frac{l_i l_i}{g_i}\right] - \left[\frac{a_i^{(1)} l_i}{g_i}\right]^2 Q_{d_1 d_1} - \Big\{ \left[\frac{h_i^{(1)} h_i}{g_i}\right] Q_{h_1} + \\ + \left[\frac{h_i^{(2)} l_i}{g_i}\right] Q_{h_2} + \left[\frac{h_i^{(3)} l_i}{g_i}\right] Q_{h_3} \Big\}^2 - \Big\{ \left[\frac{s_i^{(1)} l_i}{g_i}\right] Q_{s_1} + \\ + \left[\frac{s_i^{(2)} l_i}{g_i}\right] Q_{s_2} + \left[\frac{s_i^{(3)} l_i}{g_i}\right] Q_{s_3} \Big\}^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (13)$$

In this the quantities $a_1^{(1)}$, $a_2^{(1)}$, $a_3^{(1)}$ are the coefficients of α_1, β_1, and γ_1, in the first triangle condition; the quantities $h_i^{(1)}$, $h_i^{(2)}$, $h_i^{(3)}$ the coefficients of these quantities in the 3 horizon-conditions, and $s_i^{(1)}$, $s_i^{(2)}$, $s_i^{(3)}$ those in three sine-conditions.

According to (12):

$$l_1 = \frac{\partial \log \xi_1}{\partial \alpha_1} = i$$

$$l_2 = \frac{\partial \log \xi_1}{\partial \beta_1} = -\cot \beta_1$$

$$l_3 = \frac{\partial \log \xi_4}{\partial \gamma_1} = +\cot \gamma_1.$$

For abbreviation a symbolical way of writing has been used in (13) for the weight values. By an expression as:

$$(a\,Q_1 + b\,Q_2)^2$$

is meant:

$$a^2\,Q_1^2 + 2a\,bQ_1\,Q_2 + b^2\,Q_2^2 = a^2\,Q_{11} + 2a\,bQ_{12} + b^2\,Q_{22}.$$

The quantities $Q_{h_1 h_1}$ etc. and $Q_{s_1 s_1}$ etc. are calculated from the known equations of weight.

The coefficients of the two sets of normal equations (7) and (8) being equal, we have:

$$Q_{h_1 h_1} = Q_{s_1 s_1}\,;\; Q_{h_1 h_2} = Q_{s_1 s_2}\,;\; \ldots\ldots;\; Q_{h_3 h_3} = Q_{s_3 s_3}.$$

We may, therefore, ignore the difference between the "Q"-values with indices h and s; hence we write:

$$Q_{h_1 h_1} = Q_{s_1 s_1} = Q_{11} \quad ; \quad Q_{h_1 h_2} = Q_{s_1 s_2} = Q_{12}\,;\ \text{etc.}$$

From (5) follows for $Q_{d_1 d_1}$:

$$Q_{d_1 d_1} = g_1\ cot\,\alpha_1\ cot\,\beta_1\ cot\,\gamma_1.$$

We find further:

$$\left[\frac{l_i\,l_i}{g_i}\right] = \frac{1}{g_1}(-tg\,\alpha_1 + cot\,\beta_1 + cot\,\gamma_1) = \frac{1}{g_1}\,tg\,\alpha_1(1 - cot\,\beta_1\,cot\,\alpha_1 - cot\,\gamma_1\,cot\,\alpha_1) =$$

$$= \frac{1}{g_1}\,tg\ \alpha_1\ cot\,\beta_1\ cot\,\gamma_1$$

$$\left[\frac{a_i^{(1)}\,l_i}{g_i}\right] = \frac{1}{g_1}\,(i\ tg\ \alpha - 1 + 1) = \frac{1}{g_1}\,i\ tg\ \alpha.$$

In consequence of this:

$$\left.\begin{aligned}\left[\frac{l_i\,l_i}{g_i}\right] - \left[\frac{a_i^{(1)}\,l_i}{g_i}\right]^2 Q_{d_1 d_1} = -\frac{1}{g_1}\,tg\ \alpha_1\ cot\,\beta_1\ cot\,\gamma_1 + \\ + \frac{1}{g_1^2}\,tg^2\,\alpha_1\,g_1\ cot\,\alpha_1\ cot\,\beta_1\ cot\,\gamma_1 = 0\end{aligned}\right\}. \quad . \quad (14)$$

Further:

$$\left[\frac{h_i^{(1)}\,l_i}{g_i}\right] = \frac{1}{g_1}\{\,i\,(cot\,\beta_1 + cot\,\gamma_1) + cot\,\beta_1\ cot\,\gamma_1 - cot\,\beta_1\ cot\,\gamma_1\,\} =$$

$$= \frac{i}{g_i}\,(cot\,\beta_1 + cot\,\gamma_1) = i\,p.$$

$$\left[\frac{h_i^{(2)}\, l_i}{g_i}\right] = \frac{1}{g_1}(-i\cot\gamma_1 - \cot\beta_1\cot\alpha_1 - \cot\beta_1\cot\gamma_1 - \cot\alpha_1\cot\gamma_1) =$$

$$= \frac{1}{g_1}(-i\cot\gamma_1 - 1) = \frac{i}{g_1}(-\cot\gamma_1 + i) = iq.$$

$$\left[\frac{h_i^{(3)}\, l_i}{g_i}\right] = \frac{1}{g_1}(-i\cot\beta + 1) = \frac{i}{g_1}(-\cot\beta_1 - i) = ir.$$

$$\left[\frac{s_i^{(1)}\, l_i}{g_i}\right] = \frac{1}{g_1}(\cot\beta_1 + \cot\gamma_1) = p.$$

$$\left[\frac{s_i^{(2)}\, l_i}{g_i}\right] = \frac{1}{g_1}(-\cot\gamma_1 + i) = q.$$

$$\left[\frac{s_i^{(3)}\, l_i}{g_i}\right] = \frac{1}{g_1}(-\cot\beta_1 - i) = r.$$

Introducing these results into (13), we find in connection with (14):

$$Q\,log\,\xi_1, log\,\xi_1 = i^2(pQ_1 + qQ_2 + rQ_3)^2 + (pQ_1 + qQ_2 + rQ_3)^2 = 0.$$

In an analogous way it is shown that $Q_{\xi_1\eta_1}$ is zero, etc.

It is easy to see that *all* the weight values of the quantities ξ, η, and ζ must be zero, because the properties, on which this becoming zero rests, present a general character.

Thus the proof has been furnished that the error-curve of every point of the net is a circle.

4. *Properties of the mean errors of angles and sides of the net.*

We consider one of the quantities ξ, η, ζ, e. g.

$$log\,\xi_1 = log\,sin\,\gamma_1 - log\,sin\,\beta_1 + i\,\alpha_1.$$

When the law of propagation of the errors is applied to this then:

$$Q\,log\,\xi_1, log\,\xi_1 = (\cot\gamma_1\, Q_{\gamma_1} - \cot\beta_1\, Q_{\beta_1} + i\, Q_{\alpha_1})^2 \text{ must be } = 0,$$

or by putting the real and the imaginary parts of this $= 0$:

$$\cot^2\gamma_1\, Q_{\gamma_1\gamma_1} - 2\cot\gamma_1\cot\beta_1\, Q_{\beta_1\gamma_1} + \cot^2\beta_1\, Q_{\beta_1\beta_1} - Q_{\alpha_1\alpha_1} = 0$$

and:

$$\cot\gamma_1\, Q_{\alpha_1\gamma_1} - \cot\beta_1\, Q_{\beta_1\alpha_1} = 0$$

or:

$$Q_{\alpha_1\beta_1} : Q_{\alpha_1\gamma_1} = \cot\gamma_1 : \cot\beta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Thus from the weight value of $log\ \eta_1$ may be derived:

$$Q_{\alpha_1\beta_1} : Q_{\gamma_1\beta_1} = cot\ \gamma_1 : cot\ \alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

From (17) and (18) follows, therefore, that we may write, when K is a factor of proportion:

$$\left.\begin{aligned} Q_{\beta_1\gamma_1} &= K\ cot\ \alpha_1 \\ Q_{\gamma_1\alpha_1} &= K\ cot\ \beta_1 \\ Q_{\alpha_1\beta_1} &= K\ cot\ \gamma_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Further after adjustment α_1, β_1 and γ_1 will satisfy:

$$\alpha_1 + \beta_1 + \gamma_1 = 180^\circ$$

hence:

$$Q_{\alpha_1\alpha_1} + Q_{\alpha_1\beta_1} + Q_{\alpha_1\gamma_1} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

(This is nothing but the correlation value of the quantities α_1 and $\alpha_1 + \beta_1 + \gamma_1$, the latter of which is errorless. Hence the correlation value is zero).

From (19) and (20) follows:

$$Q_{\alpha_1\alpha_1} = -K\ (cot\ \gamma_1 + cot\ \beta_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Analogous expressions are found for $Q_{\beta_1\beta_1}$ and $Q_{\gamma_1\gamma_1}$.

From (19) and (21) it is concluded that in every triangle we need only calculate one of the 6 weight- or correlation-quantities in the usual way to be able to find the 5 others from these formulae.

From (21) follows besides:

$$Q_{\alpha_1\alpha_1} : Q_{\beta_1\beta_1} : Q_{\gamma_1\gamma_1} = \frac{sin\ \alpha_1}{sin\ \beta_1\ sin\ \gamma_1} : \frac{sin\ \beta_1}{sin\ \alpha_1\ sin\ \gamma_1} : \frac{sin\ \gamma_1}{sin\ \alpha_1\ sin\ \beta_1}$$

$$= sin^2\ \alpha_1 : sin^2\ \beta_1 : sin^2\ \gamma_1.$$

As the mean errors in α_1, β_1, and γ_1 are to each other as the roots from the weight values, we have:

$$m_{\alpha_1} : m_{\beta_1} : m_{\gamma_1} = sin\ \alpha_1 : sin\ \beta_1 : sin\ \gamma_1,$$

hence *after the adjustment the mean errors of the three angles of a triangle of the net are to each other as their sines or as the lengths of their subtending sides.*

When we write:

$$log\ \xi_1 = log\ sin\ \gamma_1 - log\ sin\ \gamma_1 + i\ \alpha_1$$

as:

$$log\ \xi_1 = A_1 + i\ \alpha_1,$$

in which A_1 represents the logarithm of the proportion of the lengths of the sides 1—2 and 1—3 of $\triangle$ 123 (fig. 1), the law of propagation of the errors yields:

$$Q_{A_1 A_1} = Q_{\alpha_1 \alpha_1}, \; Q_{\alpha_1 A_1} = 0.$$

We have, therefore, the thesis:

After adjustment the mean error of an angle of a net is equal to the mean error in the logarithm of the proportion of the circumjacent triangle sides.

The properties of the mean errors of the angles and the sides of a triangle of the net developed in this §, are valid for every triangle formed by 3 points having each a circular error-curve. This property may be proved by proving for such a triangle the existence of relations analogous to (17) and (18), which can be done by expressing the elements of the triangle which appear in it, in the coordinates of the angle points.

5. *Conformable connection of a net with circular error-curves.*

If it is desired to connect the net of triangles of fig. 1 conformably to an arbitrary number of points of a net of higher order, the error-curves of the points will remain circles, if the points in the net of higher order have also circular error-curves.

Let the net of fig. 1 be connected to the points 4, 6, and 9. The coordinates of these points in the net are:

$$z_4 = x_4 + i y_4, \; z_6 = x_6 + i y_6, \; z_9 = x_9 + i y_9.$$

In the net of higher order the coordinates of these points are:

$$Z_4 = X_4 + i Y_4, \; Z_6 = X_6 + i Y_6, \; Z_9 = X_9 + i Y_9.$$

The coordinates of the connected point z then follow from the formula:

$$Z = Z_4 \frac{(z_6 - z)(z_9 - z)}{(z_6 - z_4)(z_9 - z_4)} + Z_6 \frac{(z_4 - z)(z_9 - z)}{(z_4 - z_6)(z_9 - z_6)} + Z_9 \frac{(z_4 - z)(z_6 - z)}{(z_4 - z_9)(z_6 - z_9)}$$

In general we have, therefore:

$$Z = F(Z_1, Z_2, Z_3, \ldots . Z_n, z_1, z_2, \ldots . z_n)$$

in which Z_l are the direction values of the points in the primary net, and z_l those of the points in the secondary net.

If now the error-curves of all the points are circles, then the weight value of Z must likewise be zero, by which the property has been proved.

In the connected net exist the properties of the Q-values, therefore, also again.

Chemistry. — *On the isolation of a toxic bacterial pigment.* (Provisional communication.) By A. G. VAN VEEN and W. K. MERTENS. (Communicated by Prof. P. VAN ROMBURGH.)

(Communicated at the meeting of June 24, 1933).

In the province of Banjumas (Central Java, Netherlands Indies), which has a population of ± 5.000.000 souls, there have since time immemorial occured repeatedly wholesale poisonings, caused by the consumption of different products, made, sometimes by means of fungi, from coconuts. As a rule these products are harmless, but if they are poisonous the consuming of very little of the matter (5—20 g by estimate) is already sufficient to make the poisoning fatal. It presents a very characteristic syndrome. As a rule the patient feels perfectly normal during the first few hours or even days after eating the food, then ensue dizziness, headache, pains in the stomach-region, after which finally follows faintness, either attended or not by spasms, which after a shorter or longer time, end in death, as a rule. Pathological changes in the tissues are not to be observed. The number of cases ending in death is very numerous. Outside the province of Banjumas similar poisonings do not occur, as far as we know. The population are perfectly aware of the risk they run in eating these coconut products, but they take it with a certain fatalism. The most dangerous foodstuffs are, as we found : *Bongkrek*, a product prepared by means of fungi (Rhizopus-species) from coconut-presscakes made by the population themselves, and *Semaji*, prepared from grated coconut, after the oil has been pressed out. These foodstuffs are much partaken of in Banjumas, and in fact represent prepared by-products of the coconut oil home-industry. Many theories have been propounded about these mysterious poisonings, which by the population themselves are as a rule ascribed to spirits, but the cause could not be discovered so far. A thorough investigation was made, which lasted about a year and a half, and which took place both in the laboratory and in the country concerned. Many difficulties had to be surmounted, but finally we succeeded in pointing out, that the poisoning was not caused by the appearance of pathogenous bacteria, or by the mixing with poisonous matter, either accidentally or not. As a rule it is very difficult to lay hands on even a small quantity of the usually, already very mouldy or rotting poisonous material, as the population prefer to hide it or throw it away, and since it very soon loses its poisonousness too. We succeeded however in isolating certain bacteria from the great number of fungi and bacteria of bongkrek, which bacteria have the

power to produce the deadly poison in semaji and bongkrek. These bacteria have a number of peculiarities, they are very mobile, and form long threads when cultivated on a medium containing ammonia-salts. There are several fluctuating forms of colonies, which differ but slightly from many other bacteria living on the ground or in the water. On different culture-media the bacteria, in spite of their prosperous growth, finally nearly lose their power to form poison, which however returns after oculation into a favourable medium. The poison-containing colonies are as a rule yellowish, the others white or colourless. We succeeded in finding a method quickly to indentify the bacteria, independent of their power to create poison. As suitable media for the bacteria and at the same time as a good source of poison we used at first praebongkrek-agar (praebongkrek we call the bongkrek before it is inoculated with fungi), sterilized praebongkrek and also pressed and sterilized grated coconut.

Good test-animals proved to be monkeys, which could be fed by means of a sound, pigeons, which could easily be stuffed, and young rats, to which were administered intraperitoneal injections of extracts adapted to this purpose. Monkeys show about the same symptoms as man and it was found that the poison attacks somewhere in the brains. Pigeons show polyneuritic-like symptoms, after a latent period of sometimes 24 hours, rats show terrible spasms, severe dyspnoea and cyanosis.

A condition for the good formation of poison is a large watery surface, exposed to the air, as is the case with grated coconut, with the very porous bongkrek and also with thin layers of agar containing media. On such a surface the bacteria can rapidly multiply, and consequently rapidly produce poison. This is necessary, because under the influence of air, water and other micro-organisms the poison is quickly decomposed. As a matter of fact bongkrek or semaji may become poisonous within 24 hours. In a medium rich of fat, as is the case with the above mentioned nutriments, the poison is rather stable, even when heated; thence even heated up or short-boiled bongkrek may be very poisonous.

The attempts to purify the poison to a certain extent were at first negative. To be sure the poison may be extracted from poisonous bongkrek and the other foodstuffs by means of alcohol, but on further purification inactivity would ensue as a rule and the poison would sometimes remain in one fraction, sometimes in an other. Sometimes it would only dissolve well in fat-solvents, sometimes in water etc.

Originally we thought that we had to deal with a degradation-product of some phosphatide- or cerebroside-like matter, which occurs in large quantities in bongkrek, but after some experimenting this conception proved to be incorrect. Nor was the poison to be obtained in an anything like pure state from inoculated praebongkrek. After several failures we resolved to try and make culture-media containing several constituents

of bongkrek or rather praebongkrek, and to trace from which constituent the poison is formed by the bacteria. This proved to be the fat-fraction, which, if slowly, is hydrolised by the bacteria, after which the poison is formed from the glycerol thus liberated. From very different fats and fatty oils the bacteria can thus synthesise poison. Of the many fat-containing foodstuffs however, eaten by the Javanese, practically only semaji and bongkrek, both prepared from coconut, offer favourable conditions for the growth of these bacteria and for quick production of poison. On native products prepared similarly from arachis-nuts and soja-beans the bacteria can keep alive, but can form no or not sufficient poison. Some of these products however are occasionally mixed lightly with grated coconut, and in that case they can also cause serious poisoning. If these fatty oils from arachis-nuts and soja-beans are however emulgated with agar and other necessary materials and poured out in thin layers, a strong formation of poison takes place after inoculation.

From agar media however, which contain coconut oil or glycerol as sources of poison, the poison can not by any means either be gained pure after extraction with alcohol. This is prevented by the degradation-products of the agar. At last we proceeded to cultivate the bacteria in large flat vessels provided with a double layer of filter-paper soaked in a glycerol-containing food-solution. After about 40 hours a maximal quantity of poison has already been formed. The formation of poison both on agar-containing media and agarless media is always attended by the appearance of a yellow colour. As a matter of fact the poison itself is an intense yellow, very often however there is also at least one other yellow, inactive pigment present.

From these intensely yellow cultures, made on filter-paper, we could isolate the poison in more or less pure, if amorfous condition, by means of experiments with small quantities only. It proved however to be an extremely active substance, which, when exposed to water and air at a higher temperature, rapidly decomposes. This is the reason why we absolutely failed to be able to isolate larger quantities. If we wanted successfully to try and isolate the poison in larger quantities and, if possible, in crystalline state, we had to avoid any rise of temperature, air and dampness had to be eliminated as far as possible and we had to work quickly. It is very difficult in tropical countries to comply with all these conditions, especially when it is impossible to work in a neutral atmosphere through lack of nitrogen or hydrogen. Even the distilling of solvents in vacuo had to be avoided as much as possible.

Finally we succeeded in finding a method that was both simple and effective and which was based on the fact, that in water the poison dissolves very easily, in a saturated solution of sodium sulphate however with great difficulty. Starting from a 200 cc solution of 1 $^0/_0$-glycerol which moreover contains $^1/_2$ $^0/_0$ pepton and the necessary nutritious salts,

one obtains after about 40 hours a deep yellow liquid, which is sufficient to kill 400—800 rats within a few hours (after intraperitoneal injection of an extract prepared for this purpose). The liquid which during these 40 hours has already partly evaporated, is cooled with icewater and saturated with anhydrous sodium sulphate and at the same time shaken out with chloroform. Thus much inactive yellowish brown matter and albuminous substance is practically got rid of. Next lowboiling petroleum-ether is added to the chloroform and the whole is shaken out with small quantities of cold water, after which the poison rather easily passes into the latter, whereas the colour serves as a measure. This aqueous solution (about 20—30 cc) is once more saturated with sodium sulphate while being cooled with ice, and now shaken out with ethyl acetate. Bacterial fat and carbohydrates have now practically disappeared. A little petroleum-ether is once more added to the ethyl acetate and it is once more shaken out with as little water as possible. Thus we usually obtained about 5 cc of a deep-yellow liquid (greenish) which as a rule contained sufficient poison to kill 300—700 rats and which had a dry weight of 10—20 mg. The then following manipulations take place at the lowest temperature possible and then as much as possible in vacuo.

These operations will be more circumstantially described elsewhere, they mostly aim at eliminating the inactive matter which does not easily dissolve in water and ethyl-acetate, whereas petroleum-ether and ether dissolve even other impurities.

Finally we obtained a more than 75 % yield (= 2 à 3 mg!) of a quantity of yellow needle-shaped crystals, melting sharply at about 200°. They are insoluble in petroleum-ether, hardly soluble in ether, more easily soluble in benzene, chloroform and ethyl-acetate, very soluble in water, alcohol and fats. Very much diluted solutions are greenish yellow, more concentrated ones yellow. The yellow colour is very intense; even in a dilution of 1 : 200000 it is still to be observed. There shows a faint green fluorescense which is increased by dissolving in amyl-alcohol. The substance is rather labile; when crystallised from warm benzene part already becomes oily, so it is very difficult to get it pure enough for analysis. It contains much nitrogen, shows no alcaloid reactions, has an amphoteric character and is easy to dialyse. When treated with a saturated solution of bisulphite a beautiful brickred additional-compound of the poison is precipitated. The substance is quickly discoloured by sulphite in a slightly acid or alcaline watery solution; by shaking with air (if necessary after acidifying), or by adding a solution of bromine the yellow colour immediately returns. When treated with mineral acids the yellow colour is preserved rather a long time, on alcali being added however it disappears quickly. If in this latter case acidifying again takes place, immediately the yellow colour returns, but if this is done after an interval the solution remains colourless. The poison is very constant with regard to solution of bromine, even at 100° the activity is preserved, so the yellow colour

is not to be ascribed to ethenoid linkages. Its stability at different p_H's in regard to light has not yet sufficiently been investigated. The visible spectrum beneath 450 $\mu\mu$ is almost entirely adsorbed, the one between 450-500 $\mu\mu$ partly. The activity is very high, 5 γ (0,005 mg) administered intraperitoneally to a rat kills it within an hour. For a monkey the deadly dose will be about 50-100 γ, administered per os, for a man by estimate $^1/_2$ mg if the poison appears in its most active state. For it has been proved that during the process of purification of the poison it becomes less active when administered per os. If however it is adsorbed on caseïne or fatty acids in a certain manner, it becomes highly active again. The action of the extremely purified poison, when injected intraperitoneally, is quicker than that of the raw product, but lasts much shorter. If the bacteria form the poison from fats rather than from free glycerol the poison proves to be partly linked to fatty compounds, in consequence of which its solubility is greatly altered and the substance seems to be much more stable in the organism. So in this respect too there are many interesting problems.

As to the mode of action of this extremely active brain-poison, we are of opinion that the substance which can so very easily be reduced and oxydised reversibly, may possibly interfere with the oxydation-reduction systems of the brains. In this respect it is of importance, that the poison seems to be closely related to the yellow pigment of the second "Atmungs-Ferment" of WARBURG[1]) and seems to belong to the new class of pigments which R. KUHN comprises under the name of "Flavins". To this class the vitamine B 2 may also belong according to KUHN,[2]) together with the "Cytoflav" from the cardiac muscle and the "Lactochrom" of milk.

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Central Medical Laboratory. Batavia, Netherlands Indies.

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[1]) Bioch. Z. **258**, 496 (1933) and former publications.

[2]) B **66**, 317 (1933).

Chemistry. — *The osmotic pressure and the viscocity of nitrocellulose solutions.* By E. H. BUCHNER and H. E. STEUTEL. (Communicated by Prof. A. F. HOLLEMAN).

(Communicated at the meeting of June 24, 1933).

In a previous investigation [1]) the osmotic pressure and the viscosity of acetocellulose solutions were measured. It was found that the molecular weights of different samples did not vary so strongly as might have been expected from the very different values of the viscosity. It was concluded that the very simple relation between molecular weight and viscosity given by STAUDINGER, did not agree with the facts.

It seemed desirable to carry out an analogous systematic investigation with an other substance. Nitrocellulose was chosen, measurements of which had already been made, some years ago, by DUCLAUX [2]) and his collaborators, who had made use of the ordinary method of determining the osmotic pressure.

The osmotic pressures.

The measurements carried out by Mr. STEUTEL were made according to the method developed by VAN CAMPEN [3]) which was slightly modified: the osmometer was not made of copper, but of glass. Samples of nitrocellulose were kindly placed at our disposal by the Deutsche Celluloidfabrik in Eilenburg and by the Nobel Section of the Imperial Chemical Industries Ltd., to both of whom our hearty thanks are rendered.

As contrasted with the results obtained with the acetocellulose, the osmotic pressure of the nitrocellulose appeared not to be proportional to the concentration, even in the most diluted solutions, a fact already noticed by DUCLAUX. The molecular weight can, therefore, only be calculated when the p, c curve is extrapolated to zero concentration.

The results obtained with the English samples only will be given in this paper.

The particulars of the samples are:

	nitrogen	viscosity	colour
Hx 992	12.01%	very low	white
HL 643	12.20	↓	„
LH 470	10.92	↓	„
gun cotton 200	13.46	↓	brownish grey
blasting soluble 799	12.12	very high	„ „

[1]) BUCHNER and SAMWEL, These Proc. **33**, 749; Trans. Faraday Soc. **29**, 32.

[2]) DUCLAUX, Rev. Gén Colloïdes **7**, 386: C. R. **152**, 1560.

[3]) Rec. Trav. Chim. **50**, 915.

TABLE 1.

Temperature 25°.
Solvent: acetone.
Concentration: grams per 100 cc solution.
Osmotic pressure: cm. water.

Hx 992, (curve A)

c	P	P/c	$M_{c=0}$
4.81%	151.2	31.4	19.250
3.23	71.0	22.0	
2.17	42.6	19.6	
1.14	18.8	16.5	
HL 643, (curve B)			
3.27%	57.9	17.7	42.200
2.34	33.4	14.3	
1.64	19.1	11.6	
1.09	11.4	10.45	
LH 470, (curve D)			
3.00%	44.9	15.0	56.500
2.02	25.4	12.6	
0.94	7.27	7.7	
Guncot, (curve E)			
4.11%	67.0	16.3	96.000
2.85	34.4	12.1	
1.93	16.5	8.53	
Bl. sol. (curve F)			
0.85%	3.3	3.9	204.000

If these results are represented in a diagram (figure 1), the concentrations being taken as abscissae and P/c as ordinates, straight lines are obtained, which are not parallel to the c-axis, as would be required by the VAN

'T HOFF law. On the contrary, the graphs indicate a linear relation between P/c and c, or a function of the form:

$$P = Ac + Bc^2$$

a relation also suggested by WO. OSTWALD [1]).

From the value of P/c at zero concentration, the molecular weight

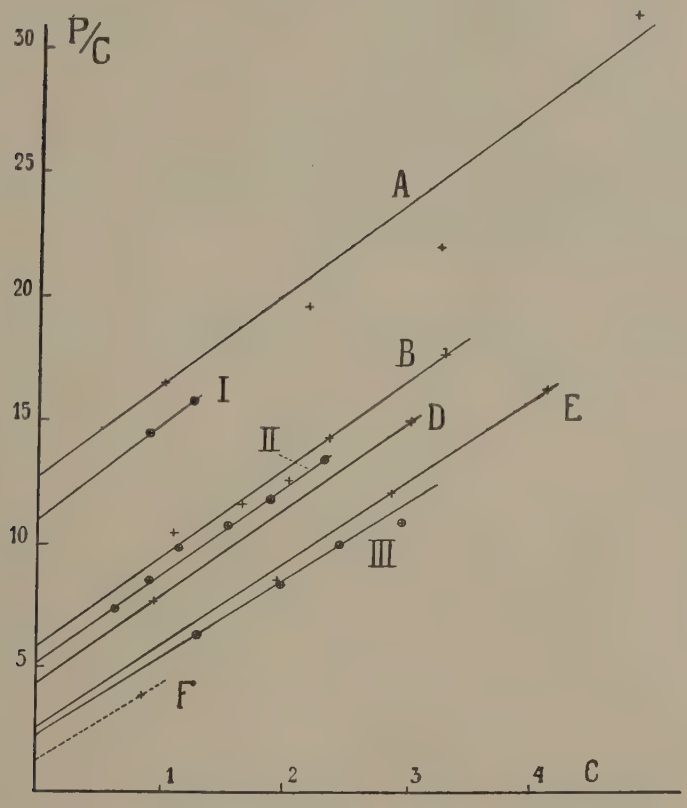

Fig. 1.
Curves A—F: STEUTEL: Curves I—III: DUCLAUX.

was calculated according to VAN 'T HOFF, with the results given in the table. It is seen that the values are widely differing in opposition to the results previously found with acetocellulose. The most striking result is, however, the parallellism of the straight lines [2]) or the constancy of the factor B. When it is considered that the concentration is expressed in grams, it will be clear that the second term of the expression for the

[1]) Koll. Z. **49**, 60.

[2]) It is worth mentioning that the experiments with the German samples show the same value of B, as do also the previous results of DUCLAUX, l.c. The latter have been inserted in figure 1.

osmotic pressure cannot depend on the number of the molecules only. Obviously the chemical nature of the dissolved substance and of the solvent play here the principal part. It is, therefore, assumed that the osmotic pressure may be divided in two parts[1]), a VAN 'T HOFF part proportional to the molecular concentration, and a second part, which represents the interaction between solvent and solute. The latter will in first approximation not depend on the degree of division of the solute: 10.000 molecules of the molecular weight M contained in V cc solvent, will show the same effect as 100 molecules of the weight 100 M.

The factor B must be expected to vary when different solutes and solvents are used: it will be a measure of the forces acting between solvent and solute. The approximate constancy of P/c for acetocellulose may, therefore, be interpreted as indicating a weaker influence of this substance on acetone, as compared with nitrocellulose, a conclusion which is in agreement with the strong polarity of the nitro-groups.

The viscosities.

The viscosities of the solutions have been determined with the ordinary OSTWALD apparatus at 25°. The results are contained in the following table.

Hx 992.		HL 643.	
c	η_r	c	η_r
4.81%	10.12	3.27	33.9
2.40	3.45	1.64	7.43
1.20	1.85	0.82	2.85
0.60	1.347	0.41	1.68
0.30	1.152	0.205	1.281
0.15	1.075	0.102	1.115

LH 470		Gun. cot.	
c	η_r	c	η_r
1.68%	11.84	1.40%	15.53
0.84	3.65	0.70	4.48
0.42	1.85	0.35	2.11
0.21	1.353	0.175	1.44
0.10	1.157	0.088	1.194

[1]) cf. WO. OSTWALD, Koll. Zschr. **24**, 7 who takes the second term to represent a swelling pressure.

Blast. sol.	
c	η_r
0.61%	23.7
0.30	5.34
0.152	2.30
0.076	1.49
0.038	1.213

If $log\ \eta$ is represented in a diagram (fig. 2) as a function of the concentration, a bundle of affined curves is obtained: these curves are nearly

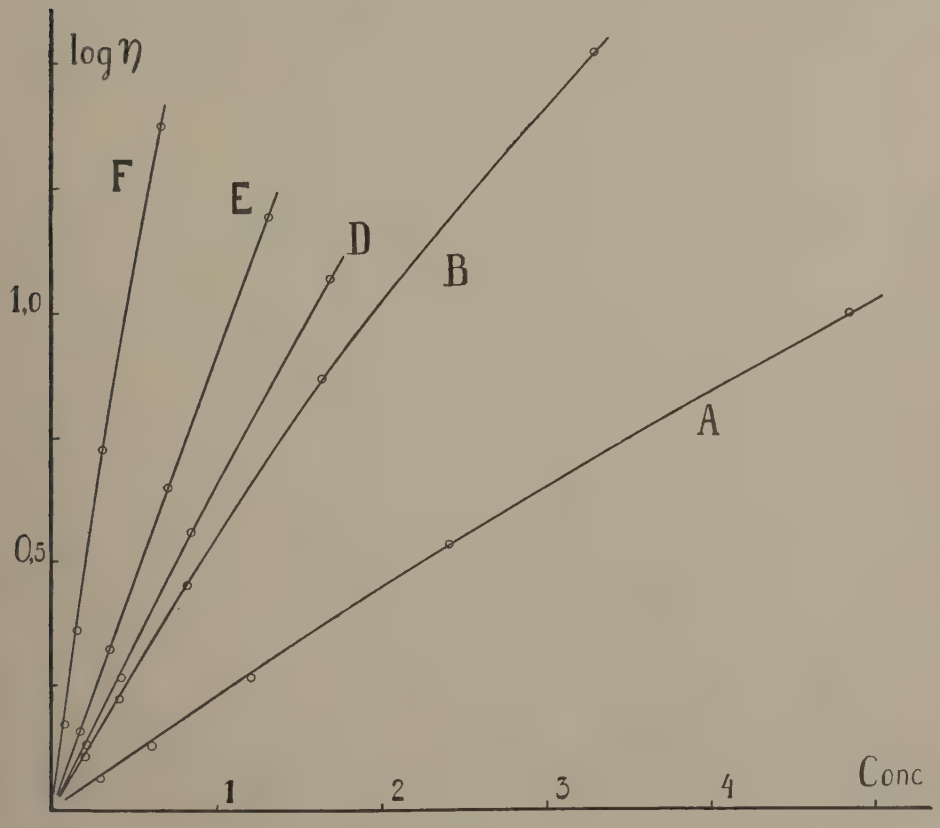

Fig. 2.

straight lines. When the concentration is calculated, at which the different sols show equal viscosity, "blast. sol." being taken as unity, nearly constant values are obtained for different values of $log\ \eta$.

log η_r	conc. Hx 992	conc. HL 643	conc. LH 470	conc. gunc	conc. bl. sol.
0.2	11.6	4.7	4.0	2.8	1
0.3	10.9	4.5	3.8	2.7	1
0.6	11.2	4.5	3.7	2.65	1
0.9	11.0	4.3	3.6	2.5	1

These values may be called the concentration values of isoviscous solutions.

This result indicates that only one factor is responsible for the differences in the viscosity of various samples. Although it was maintained in the previous communications that no simple relation between viscosity and molecular weight existed, it was also recognised that, broadly speaking, the higher viscosities corresponded to greater molecular weights. It was therefore tried whether the molecular weights, as determined by the osmotic pressure, might be brought in relation to the concentration values. It was found that the formula: conc. value $\times$ mol. weight $=$ const. holds good. In figure 3 the conc. values are plotted against the values

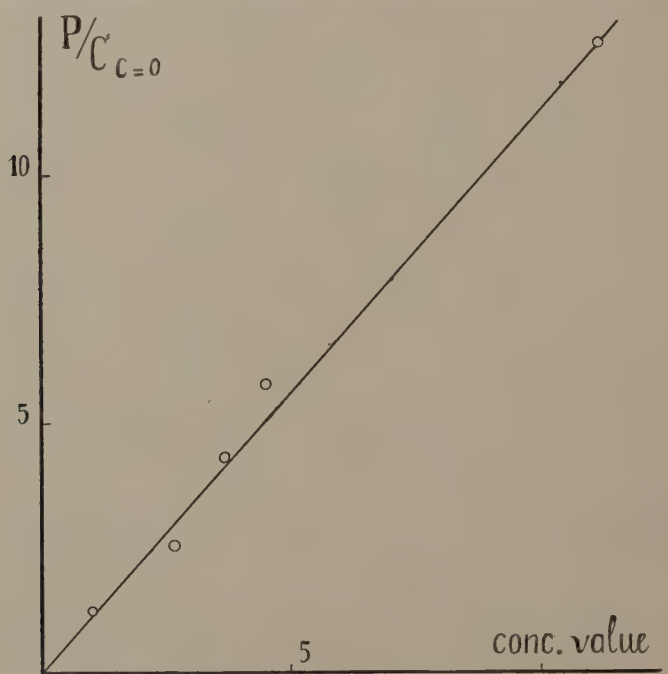

Fig. 3.

$(P/c)_{c=0}$, which are evidently proportional to the reciprocals of the mol. weights. The relation appears to be a linear one [1]).

[1]) A different formula connecting the concentrations of the isoviscous solutions with the molecular weights has been given by FIKENTSCHER and MARK, Koll. Zschr. **49**, 135. It could not be definitely tested, since the molecular weights of the samples used by these authors were not independently determined.

It will be of interest to investigate whether a similar relation holds for other series of analogous products: if so, it would indeed be possible, as STAUDINGER contends, to calculate the molecular weight from viscosity measurements. It even appears that STAUDINGER's formula may be derived — for extremely diluted solutions — from the general one which has been put forward above.

The investigations are being continued. An extensive communication will follow elsewhere.

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Geology. — *On the relation between the volcanic activity in the Netherlands East Indies and the belt of negative gravity anomalies discovered by* VENING MEINESZ. By B. G. ESCHER. (Communicated by Prof. F. A. VENING MEINESZ).

(Communicated at the meeting of June 24, 1933).

In 1912 G. A. F. MOLENGRAAFF for the first time distinguished in the East Indian Archipelago two stable regions and a labile region in between with a strong relief of the sea bottom (bibl. 1, p. 131).

This opinion was strikingly confirmed by the distribution of the epicentra of the earthquakes that were registered between 1909 and 1925 at Weltevreden and in Malabar, the position of which were determined by S. W. VISSER (bibl. 2). One stable region is formed by the continuation of the Asiatic continent in the China sea, Sumatra, the Malay peninsula, Borneo, the Java sea and Java. The second stable region is formed by Australia, the Arafoera sea and the southern portion of New Guinea. The labile region not only comprises the area between the line of Wallace and the Arafoera sea, but also a belt of the Indian Ocean to the south-west of Sumatra and south of Java, a belt to the east of the arc Minahassa-Eastern Mindanao and a belt to the north of New Guinae. To these labile belts must also be counted parts of Sumatra and Java along the Indian ocean and the northern part of New Guinea (see fig 1).

Our knowledge of this part of the earth's crust was greatly increased recently by the investigation of the force of gravity by VENING MEINESZ (bibl. 3—7). It is of great importance that the principle belt of negative anomalies of the force of gravity coincides with a large part of the epicentres of earthquakes as determined by VISSER. It is especially on New Guinea that part of the epicentres of severe earthquakes fall outside the belt of anomalies, so far as it is yet known, but the gravimetrical survey has not yet been carried out here. The concentration of earth-

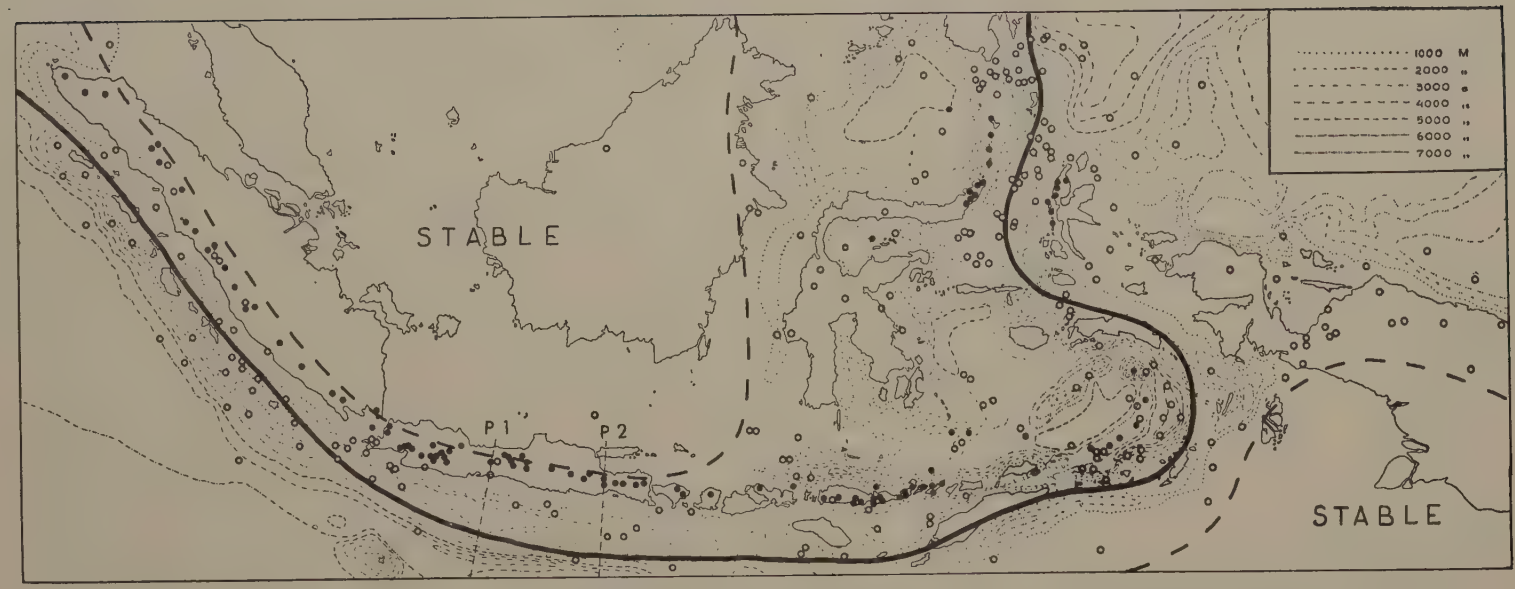


Fig. 1. Map of the Netherlands East Indian Archipelago with Active Volcanoes, Epicentra of Earthquakes and the Axis of the belt of Negative Anomalies of the Gravity.

P_1 indicates the situation of a former section, P_2 that of the section fig. 2.

quakes in the belt of VENING MEINESZ proves that a final adjustment has not been obtained here, but that the belt is in a state of motion.

The Volcanological Survey in the East Indies has ascertained the existence of 109 active volcanoes in the Netherlands Indies (bibl. 8 and 9). Of these, 86 are situated on a belt that begins in Atjeh, that continues over Sumatra, Java, Bali, Lombok, Soembawa, Flores to Lomblen, where it is discontinued on Alor, Wetar and Roma, and finally follows the row of six volcanic islands: Damar, Teoen, Nila, Seroea, Manoek and Banda-Api. The axis of this belt lies at a distance of some 100 to 250 kilometers from the axis of VENING MEINESZ.

The second region of active volcanism in the Netherlands Indies is divided into two belts on both sides of the axis of VENING MEINESZ. Ten active volcanoes lie on the western row that stretches from the Minahassa in the south to Awoe in the north, eight lie on the eastern row from Makian in the south to the volcanoes in northern Halmahera in the north. The axis of VENING MEINESZ runs in between at a distance of 100 kilometers from both rows.

Of the 109 active volcanoes $86 + 10 + 8 = 104$ can be counted to three groups, that are situated at a distance of 100 to 250 km from the axis of VENING MEINESZ. Of the remaining 5 active volcanoes two belong to submarine eruption points reconstructed from ship's journals and therefore not established with absolute certainty. The three others are the volcanic island Api, north of Wetar, Oena-Oena in the Gulf of Tomini and a submarine volcano west of Awoe.

The parallelism of the rows of volcanoes and the belt of negative anomalies is so striking, that some causal nexus between the two types of disturbed belts of the earth's crust must be assumed. The distance between the two categories of belts is so large, however, that no direct connection between the active volcanism and the shortage in the force of gravity can exist. It appears more likely that the two phenomena have a common cause.

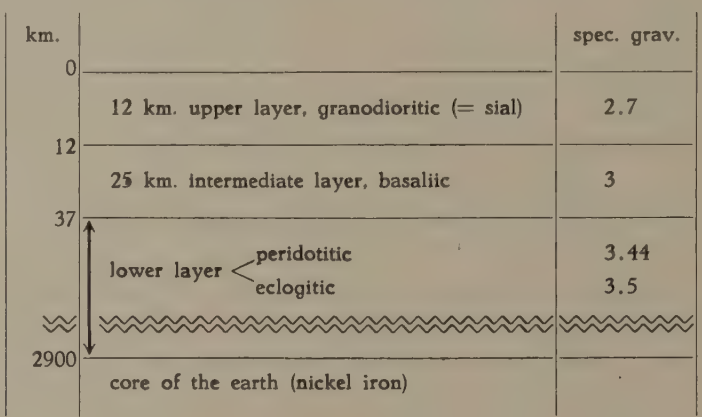
In September 1931 I pointed to such a common cause (bibl. 10) in the currents of the substratum, that lies below the earth's crust, thus making use of the idea's of AMPFERER (bibl. 11 and 12) and SCHWINNER (bibl. 13), after already having given as my opinion in 1922 that currents of magma are the common cause of mountain forming, earthquakes and volcanism (bibl. 14).

It was not until after 1931, that I came to know the very important work of ARTHUR HOLMES on currents in the substratum (bibl. 15, 16 and 17). He was the first to point to a motor that causes these currents to flow. According to HOLMES they are convection currents, that owe their origin to unequal radioactive heating. Besides planetary circulation in the substratum, he distinguishes currents, that may be compared with the monsoons and that he attributes to more intensive heating beneath the

continents, below which more radioactive substances are present than under the oceans.

The principle features are, that the warm currents ascend and flow outwards centrifugally below the continents. This causes on the one hand a drifting of the continents, by which the Atlantic Ocean with the Mid-Atlantic ridge were formed, on the other hand compression along the edges of the continents, by which marginal folded mountains are produced. Java also belongs to the marginal area of compression of Asia.

In the following I accept the subdivision in spherical shells proposed by HOLMES for the outer parts of the earth (bibl. 15). Apart from the sediments, that show considerable local variations in thickness, HOLMES distinguishes the following spherical shells below the continents, while under the oceans the sial is absent, or is only very thin.



Where continental and oceanic currents meet in the substratum, the earth's crust, that is carried along, is piled up. This piling up causes a thickening of the edge of the continent, bij which the supply of heat is gradually increased. This in turn results in the forming of a vortex that exerts a horizontal tension on the crust, and at the same time at the side of it a horizontal compression which condenses basalt to the high-pressure facies eclogite. This eclogite being denser than peridotite sinks downwards. HOLMES illustrates the principle features of this continental drift by sections (bibl. 15, 16, 17) and also the vortices, that are of the greatest importance for our problem (bibl. 15). In all these sections the strata are consciously represented too thick as compared with the horizontal dimensions.

The section through Java and the axis of VENING MEINESZ, that I published in September 1931 was titled: "first attempt for a hypothetical section". The profile reproduced here in fig. 2, reaching to a depth of 160 km., is a second attempt, in which I took the views of HOLMES into account.

VENING MEINESZ considers that the deficit of gravity in the East Indian

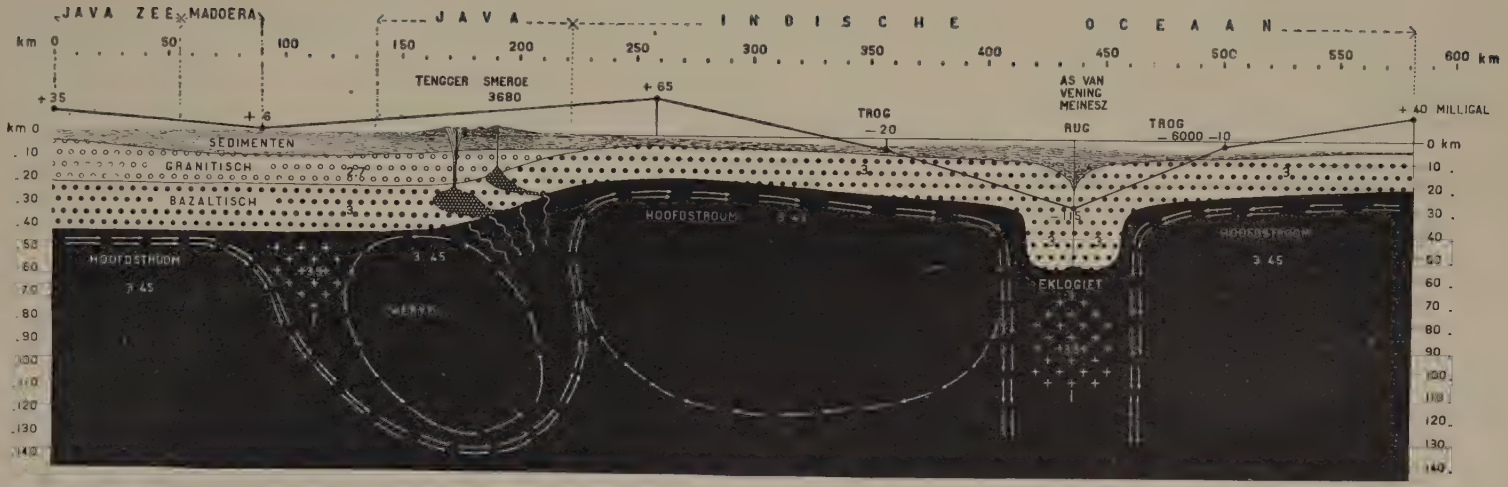


Fig. 2. Hypothetical section through Java and the Axis of VENING MEINESZ,

Scale about 1: 3.180.000

Hoofdstroom = Main current Wervel = Vortex Trog = Trench Rug = Ridge

Archipelago can only be explained by a root of the earth's crust buckled downwards, so as to force aside the heavier substratum locally. If the difference in specific gravity of 0.5 is assumed between the displacing and the displaced rocks, the vertical section of the root must possess an area of 1000 km², for instance 25 × 40 km or 20 × 50 km.

On the first of September 1931 I attributed the formation of this lighter root to two opposing currents in the substratum and four weeks later in a discussion on the earth's crust in the geographical section of the British Association, on September the 28th 1931, HOLMES quite independently said:

"The only kind of process competent to provide compression and at the "same time to exert a downward drag against gravity would seem to "involve the operation of two opposing subcrustal currents which "approach the strip and turn down beneath it" (bibl. 18, p. 447).

In my first section a vortex was drawn, because only thus could the volcanic activity on Java be plausibly explained.

In the second section the views of HOLMES are taken into account by the introduction of a general circulation (main current). The following remarks may help to explain this section:

1. The horizontal and the vertical scale are the same, both 1 : 3.180.000.

2. The topographic line both above and below sealevel was constructed from the most recent data.

3. For the gravimetrical section, expressed in milligals, sea level was taken as datum line. For this profile a section was interpolated between two sections of the force of gravity determined by VENING MEINESZ to the south of Java. The data were taken from a map of the East Indies, kindly supplied by Prof. VENING MEINESZ on which were marked the corrected values for the anomalies he determined.

4. Below Java the existence was supposed of a layer of sial with the specific gravity of 2, 7, that is absent on the bottom of the Indian Ocean.

5. Sediments occur above the sial. According to RUTTEN the thickness of the tertiary strata of Java exceed 7 km. in some places (bibl. 19, p. 96). In the section the total thickness of the sediments was represented with a maximum of 10 km., the remaining parts being given a thickness roughly estimated from the gravimetrical section.

6. The thickness and depth of the upper margin of the intermediate layer with a basaltic composition and a specific gravity of 3, were made to correspond roughly with the gravimetrical section.

7. The specific gravity for a substratum of peridotitic composition would be 3.45, sothat the difference in density between the displacing and the displaced rocks in the root is 0.45. The root of VENING MEINESZ is thus formed by the intermediate layer in our section.

8. The currents in the substratum are divided in main currents, denoted by double arrows and vortices, denoted by single arrows. We must suppose, that at first only the main currents existed, of which

the continental current was the stronger. In consequence of the meeting and plunging downwards of these principle currents the protuberance in the intermediate layer was formed, while the sedimentary strata above it were crumpled into folds.

The piling up of sial beneath Java caused a gradual increase in the supply of heat and finally produced a vortex, that exerted a contrary drag on the earth's crust below Java.

I believe that from these principles at last a plausible explanation can be deduced for the parallelism of the volcanism to, and in, folded mountain chains.

It has already for a long time been clear, that volcanism manifests itself in regions where tangential tension occurs. Iceland, where the most violent manifestations of volcanism in the form of fissure eruptions, take place, may be cited as example. There it is through tectonic tension-fissures that the magma is produced. The Laki-eruption of 1783 is the greatest known volcanic eruption of historic times.

It has always been an unsolved problem, why folded mountains are also accompanied by volcanic activity. The Circum Pacific zone is the most striking example of this combination. Folding is produced by tangential compression in the earth's crust, but this agent is not apt to allow passage for the magma through the earth's crust. It is known, that during the orogenetic proces injections of magma occur along thrusting planes, that probably act as lubricant of the tectonic movements. Basic intrusives, greenstones, ophiolites, pietre verdi, have played this part and have become known amongst others through E. ARGAND from the mesozoic envelopment of the Penninic nappes. The serpentine of the Gornergrat is an example. But it is not probable, that these injections have formed volcanic chambres, considering their slight thickness and volume and their swift crystallisation.

For volcanism considerable plutonic chambres in the earth's crust are necessary and these can only be expected where tension prevails in the earth's crust. The tension, that is called forth in the anticlinal bends under a small load, is of no importance for this problem.

The vortices that gradually evolve in the main current of the substratum, as HOLMES believes, offer an acceptable explanation for the occurrence of stretching in the earth's crust and this would also render the formation of plutonic chambres more plausible, that turn to volcanic chambres during the process of crystallisation.

I will not enter further into the problem of the shapes of the volcanic chambres here. To my mind a great difficulty is, that during the whole process there is slow motion, that must result in continuous deformations.

In accordance with the views of HOLMES concerning acid and basic injections in eruptive centra (bibl. 20), I drew the top of the volcanic chambre of the Tengger mountains (basaltic) deeper than the sial, at a

depth of 23 km., that of the Smeroe (andesitic) less deep than the sial at about 15 km. below sea level.

Finally it should be pointed out, that two regions of tangential pressure occur in the section. The most important is situated in the region of the root, where eclogite is probably formed, while a secondary region of pressure is formed to the north of the vortex, that according to HOLMES will also tend to produce eclogite. The sinking eclogite is conductive to the production of a geosyncline below Strait Madoera. Here the hypotheses of HOLMES can be brought into line with the opinion of UMBGROVE (bibl. 21, p. 36 and 39), that Strait Madoera might be a geosyncline, a view already indicated by VAN ES (bibl. 22, p. 91).

In order to render the importance of the factor time evident for our problem, I have attempted in the diagram, fig 3, to reconstruct three stages in the evolution of the section under consideration.

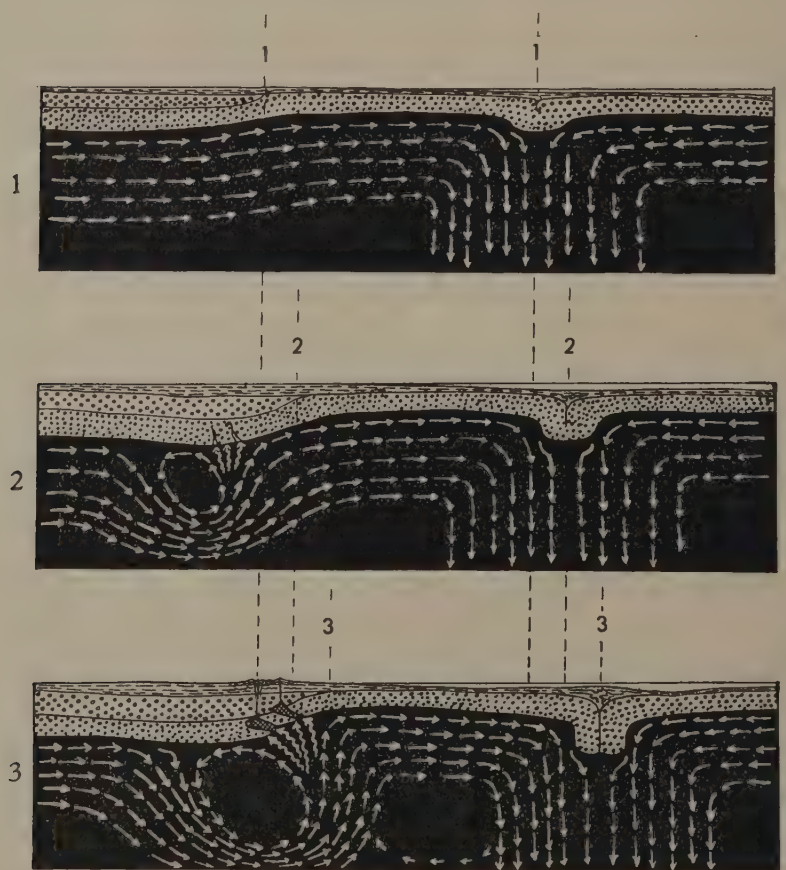


Fig. 3. Three stages in the evolution of the section through Java and the belt of negative anomalies of the gravity.

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Geology. — *Die Anwendung der Undationstheorie auf das alpine System in Europa.* Von R. W. VAN BEMMELEN. (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated at the meeting of June 24, 1933).

Im Vorliegenden wird einigen Problemen der alpinen Orogenese nachgegangen, die nach den bestehenden Theorien schwer oder nur gezwungen zu erklären sind. Dieser Rahmen lässt allerdings nur die Behandlung einiger Leitlinien des Gedankenganges zu [1]).

1. *Allgemeine Betrachtungen* (Fig. 1).

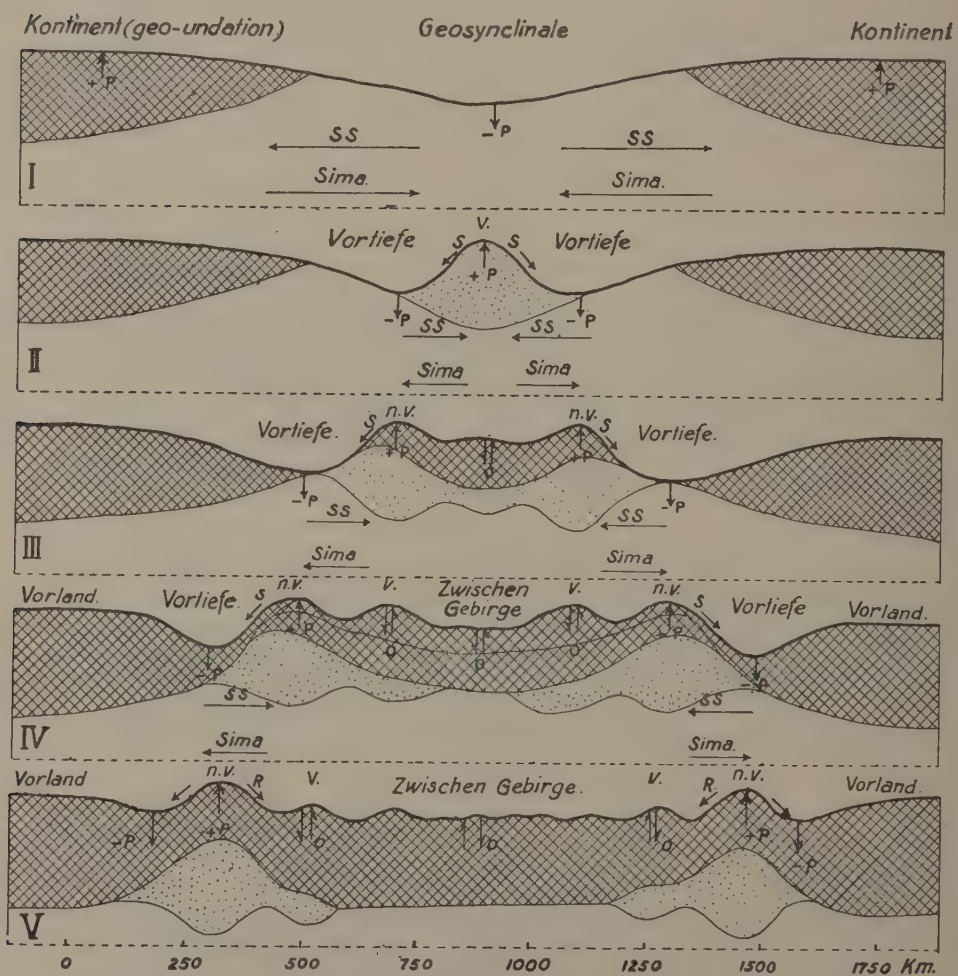

Fig. 1.

(Für Erklärung zu Fig. 1. siehe folgende Seite).

[1]) By want of space a list with litterature-references has been dropped.

ERKLÄRUNG ZU FIG. I.

Die ontologische Entwicklung eines zweiseitig symmetrisch gebauten Kettengebirges aus einer Geosynklinale nach der Undationstheorie.

I. *Geosynklinalstadium:* Geokrate Periode; Aufwölbung der Kontinente und ziemlich enge, aber tiefe Geosynklinale. Anregung des Differentiationsprozesses unter den tiefsten Teilen der Geosynklinale.

II. *Embryonalstadium:* Aufwölbung einer Mittelschwelle und Einsinken der Vortiefen. Anregung der Differentiation unter den Vortiefen.

III. *Jugendstadium*: Aus den Vortiefen entstehen Aufwölbungen, die im allgemeinen zunächst nicht vulkanisch sind, während die ursprünglichen Vortiefen weiter nach dem kontinentalen Rahmen hinwandern.

IV. *Frühreifes Stadium:* Die sich seitwärts fortpflanzenden Mesoundationen haben sich in einen vulkanischen Innenbogen und einen nichtvulkanischen Aussenbogen mit nach aussen gerichteter Faltungs- und Ueberschiebungstektonik differentiiert. Sie umgeben das Zwischengebirge, das je nach der Phase der Oszillation, in der es sich befindet ein Gebirge oder ein Innensenke ist. Im allgemeinen hat in diesem Stadium bereits eine Hemmung der Wellenbewegung durch das kontinentale Vorland stattgefunden, sodass der Bogen in seiner Form an den Rahmen angepasst wird (z.B. Molukken-bogen am australischen Vorland).

V. *Reifes Stadium:* Die Welle hat sich am Rahmen des Vorlandes totgelaufen, dabei das sekundärtektogenetisch zusammengeschoben Deckenpaket überholt und zu einem Gebirge gehoben. Die rückwärtigen Teile der Decken erfahren infolge eines entgegengesetzten Gefälles eine Rückfaltung. Die älteren kontinentalen Kerne sind schliesslich mit dem Gebirgssystem verschweisst.

Schraffiert: Die Kruste kristalliner salischer Differentiationsprodukte (mit lokalen, noch nicht erlöschenen Vulkanherden) und Sedimente.

Punktiert: Salische noch nicht kristallisierte Restlösungen (Batholithe in "statu nascendi"). Spez. Gewicht vor der Abgabe der flüchtigen Bestandteile und der Kristallisation 2,3-2,5, nach der Kristallisation 2,6-2,8.

Der Masstab des Oberflächenreliefs ist übertrieben, der Masstab der subkrustalen Grenzflächen dagegen stark reduziert.

$+p$ = Primärtektogenetische Hebung.
$-p$ = „ Senkung.
S = Sekundärtektogenese.
R = Rückfaltung.
SS = Salsima (Muttermagma, basaltisch).
$V.$ = Vulkanisch.
$n.V.$ = Nichtvulkanisch.
$O.$ = Oszillierende Bewegung.

Wie in früheren Arbeiten erörtert wurde, lassen sich die Undationen der Erdoberfläche nach ihrer Grösse in drei Gruppen scheiden.

Die *Undationen der ersten Ordnung* oder *Geoundationen* verursachen die Hebung ganzer Kontinente und die Senkung geosynklinaler Gebiete. Sie besitzen eine Spannweite von über 1000 km. Die Geosynklinalgebiete sind die Entstehungsgebiete der grossen Gebirgssysteme, wie des kaledonischen, des herzynischen und des alpinen Systems.

Die Entwicklung solcher Geosynklinalen zu Gebirgssystemen umfasst Perioden von mehreren Hunderten Millionen Jahren.

In diesen Geosynklinalgebieten entwickeln sich sodann *Undationen der zweiten Ordnung* oder *Mesoundationen*; das sind selbständige Gebirgsketten mit einer mittleren Breite von etwa 200 km.

Wie a.a.O. von mir auseinandergesetzt wurde, trachten die Mesoundationen seitwärts zu wandern. Sie verbreiten sich von einem Störungszentrum aus wie Wellen über einen Wasserspiegel. Die Periodizität der seitlichen Verschiebung der Mesoundationen beträgt 10—50 Millionen Jahre [1]).

Bei der seitlichen Verschiebung werden das äussere Krustenmaterial und die sich in den Vortiefen anhäufenden Sedimente in Form von Überschiebungen nach dem Vorland zubewegt.

An der Innenseite eines solchen Überschiebungsbogens erfährt die Kruste eine Zerrung, wodurch eine vulkanische Innenzone entsteht.

In den Mesoundationen können noch kleinere Einheiten, *Undationen der dritten Ordnung* (Detailundationen, minor-undations, Spezialundationen, STILLE unterscheiden werden; das sind dann kleinere „Tumoren" von nur einigen Dutzenden km Breite.

Will man *die Hauptzüge eines Gebirgssystems* herausschälen, so muss man von Undationen der ersten Ordnung beherrschte Leitlinien von solchen trennen, die von Undationen der zweiten und der dritten Ordnung bestimmt werden.

Das Gesamtbild eines Gebirgssystems wird vom Verlauf der Geosynklinalzone beherrscht. Wie KOBER überzeugend dargelegt hat, umschliessen die Geosynklinalzonen die älteren kontinentalen Kerne.

Der Verlauf der einzelnen Gebirgsstränge wird bestimmt durch ihre Parallelität mit den Störungszentren in der Geosynklinalzone, sowie — in einem späteren Entwicklungsstadium — durch die Anpassung an den Rahmen des Vorlands. So kann es vorkommen, dass Ketten quer zur allgemeinen Richtung der Geosynklinalzone verlaufen, und können Beugungen, Kettungen, Scharungen und Virgationen entstehen. Eine Vereinigung der Mesoundationen zu zusammenhängenden Stämmen im Sinne der meisten Geotektoniker (STAUB, KOBER, STILLE) erscheint mir nicht angebracht. Die Mesoundationen sind vielmehr im Zusammenhang mit ihren eigenen Störungszentren zu betrachten, wenn man nicht die Grössenordnung dieser Ketten aus dem Auge verlieren will. Die Wirkungssphäre der Mesoundationen ist nämlich eine Dimension kleiner als die der Geoundationen.

Um eine weitere Dimension kleiner ist sodann die Wirkungssphäre der Detailundationen, die ihrerseits *stellenweise Variationen* verursachen im Streichen der verschiedenen Gebirgsbögen, Hoch- und Tiefgebiete, Tumoren i.e.S.

Die sekundärtektogenetischen Bewegungen erfolgen nach HAARMANN (1930) in der Richtung des grössten Gefälles und die Überschiebungen stellen Abgleitungen von den primären Aufwölbungen nach den Vortiefen infolge der Schwerkraft dar. Diese Bewegungen finden darum unabhängig von den Widerständen des Vorlandes statt. Sie werden nur schliesslich durch den Vorlandrahmen aufgehalten (Branden oder Stirnen der Decken).

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[1]) Die Periodizität dieser Erscheinungen kann nach dem Prinzip der Relaxation-Schwankungen erklärt werden.

Nach der Undationstheorie brauchen darum die kontinentalen Kerne sich einander nicht zu nähern wie Backen eines Schraubstockes.

Die zentrifugale Überschiebungstektonik, wie sie die Bögen der westbetischen Kordilleren und des Riffs, sowie der Westalpen und der Ostkarpathen so deutlich zeigen, ist mechanisch einwandfrei nur zu erklären, wenn man annimmt, dass das Material vom Zentrum aus nach aussen geschoben wurde.

Eine zentripetale Unterschiebung des Vorlands, wie sie KOBER annimmt, würde eine so gewaltige Querstauung der Vorlandmasse zur Folge haben, wie sie wohl nirgends zu beobachten ist. Eine einseitige Nordbewegung des Gondwanakontinents im Sinne STAUBs kann ein derartiges Bild ebenfalls nur sehr gezwungen erklären. Auch mit dem Schlagwort STILLES, dass man es mit einem Kampf „aller gegen alle" zu tun hat ist nichts gewonnen. *Nur wenn man annimmt, dass die Decken vom aufgestiegenen Zentrum aus in die umgebende Vortiefe abgeglitten sind, dass erst daraufhin die letztere zum Gebirge erhoben wurde und das Zentralgebiet wieder zur Innensenke einsank, ist man aller Schwierigkeiten bei der Erklärung der räumlichen Entwicklung enthoben.*

Von SIEBERG wurde jüngst (1932) wieder der Nachdruck auf das *Gesetz der Volumenkonstanz* beim tektonischen Geschehen gelegt. Aus diesem Gesetz folgt, dass jeder undatorischen Hebung ein Sinken des Nachbargebietes entspricht. Die primäre Bodenoszillationen sind die Folge der Interferenz von Impulsen, die auf Undationen gleicher und verschiedener Ordnung zurückzuführen sind. Teile von Bergketten können wieder wegsinken als Folge des volumetrischen Ausgleichs in der Nähe gelegener Mesoundationen, aber auch von Geoundationen des Vorlandes. Dadurch entziehen sich Teile der Gebirgsbögen der Beobachtung und entsteht ein kompliziertes Bild von Hoch und Tief, in dem häufig die einzelnen Bögen in ihrem Verlauf schwer zu entwirren sind.

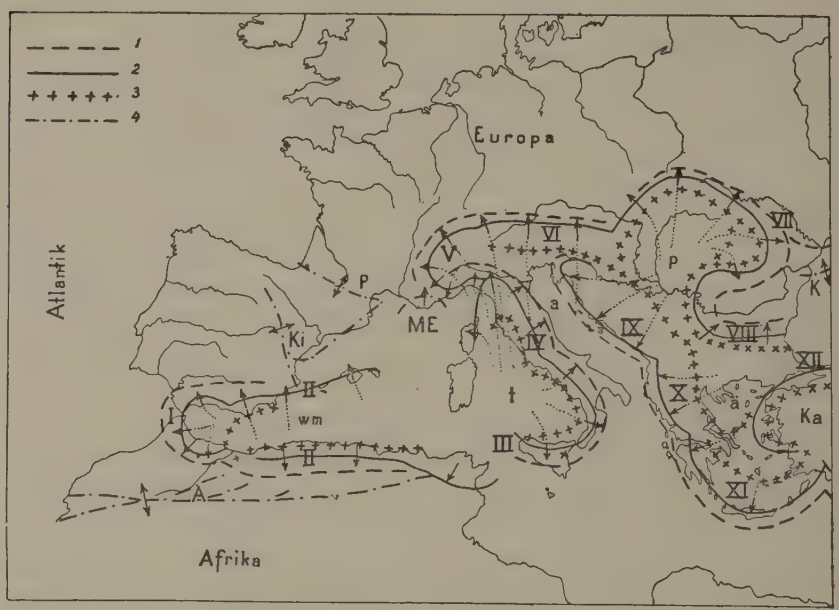
So bildet das heutige mittelländische Meer den volumetrischen Ausgleich sowohl von Hebungen des kontinentalen Vorlandes (Europa und Afrika) als von solchen der Mesoundationen im Geosynklinalgebiet selbst. Die heutige Verteilung von Hoch und Tief weicht bereits sehr wesentlich von der der jüngsten geologischen Vergangenheit ab (v. SEIDLITZ, 1932). Dies ist besonders zu beachten bei der Beurteilung der Entstehung von Gebirgssystemen mit Hilfe der Undationstheorie.

2. *Das alpine System in Europa* (Fig. 2).

Da das Gebiet des mittelländischen Meeres schon teilweise von der herzynischen Gebirgsbildung ergriffen war, besass die alpine Geosynklinale eine komplizierte Form. Es fehlte eine lange durchlaufende Medianzone, von der aus sich ein einfach zweiseitig gebautes Orogen im Sinne KOBER's hätte entwickeln können; vielmehr entstand zunächst im allgemeinen Senkungsgebiet eine Anzahl tiefer Becken. Von diesen *„zentralen Becken"* aus

entwickelten sich mehr oder weniger konzentrische, sich teilweise schneidende und überdeckende Wellen (Meso-undationen).

Es lassen sich folgende *Störungszentren unterscheiden*: das westmediterrane, das tyrrhenische, das alpine, das pannonische, das ägäische und das kleinasiatische.



ERKLÄRUNG ZU FIG. 2.

Das alpine System in Europa nach der Undationstheorie.

1. Vortiefen.
2. Nichtvulkanische Aussenbogen mit zentrifugaler Ueberschiebungstektonik.
3. Vulkanische Innenzonen.
4. Vorlandketten mit Fächerstruktur.

Störungszentren: wm = westmediterran, *t* = tyrrhenisch, *a* = alpin, *p* = pannonisch *a* = ägäisch, *ka* = kleinasiatisch.

Alpine Aussenbogen: I = Rif-westbetischer-bogen, II = Balearen-ostbetischer-algerischer Bogen, III = Südapenninbogen, IV = Nordapenninbogen, V = Westalpenbogen, VI = Ostalpenbogen, VII = Karpathenbogen, VIII = Ostbalkanbogen, IX = dalmatischer Bogen, X = albanischer Bogen, XI = hellenischer Bogen, XII = kleinasiatischer Bogen.

Vorlandketten: Ki = Keltiberische Ketten, *P* = Pyrenäen, *M-E* = Maures-Estérel, *K* = Kaukasus, *A* = Atlas.

Ausserhalb des allgemeinen geosynklinalen Senkungsgebiets entstanden zwischen den kratogenen Schollen des Vorlands auch langgestreckte Sedimentationströge, wie die der keltiberischen Ketten, der Pyrenäen, die von Maures-Estérel, sowie die des Kaukasus und des Atlas.

Das alpine Störungszentrum (penninische Trog i.e.S.) lag im Gebiet der heutigen Nordapenninen und der nördlichen Adria. Vermutlich war es

einigermassen unsymmetrisch gebaut und lagen die grössten Reliefunterschiede an seiner Südseite.

Infolge der fortgesetzten Senkung fand zuerst während der Jurazeit wahrscheinlich eine Art Volltroggleitung im penninischen Gebiet statt und darauf glitten auch noch die Sedimentpakete der heutigen ostalpinen Decken, zusammen mit Teilen des kristallinen Untergrunds, von der marginalen Südseite des Beckens über die penninische Serie. In der Kreide kam es sodann zur ersten Aufpressung der salischen Differentiationsprodukte, die sich unter dem penninischen Becken angehäuft hatten. Die sogenannte *Gosauaufwölbung* war am stärksten an der Südseite der penninischen Zone, die ursprünglich am tiefsten war.

Es folgte im Paläogen die *Flysch-* und im Neogen die *Molasseaufwölbung* und die Einsenkung der dazu gehörigen Vortiefen. Die Aufwölbungen wanderten in der Richtung des Vorlandes in der Weise, dass jeweils die ursprüngliche Vortiefe emporgedrückt und alsdann nach Norden verschoben wurde.

Durch sekundärtektogenetische Abgleitungen in diese Vortiefen wurde das alpine Deckensystem aufgebaut. *Die gewaltigen Beträge der ostalpinen Decken sind die Folge der hier für die Bewegungen besonders günstigen Umstände, da erst die penninischen Decken die Rollen bildeten, worüber sie nordwärts glitten, und darnach die noch unverhärteten Sedimente der helvetischen Decken.*

Der Molassetrog schliesslich wird an seiner Nordseite vom kratogenen Vorland begrenzt und kann nicht weiter „immigrieren". Er wird mit Abtragmaterial von Stellenweise 3000 m Mächtigkeit angefüllt, wogegen dann die Decken stirnen, und an der Nordseite dieses Trogs erfolgte die epidermale Jurafaltung.

Die Decken sind nicht mehr imstande, von der Aufwölbung in die Vortiefe abzugleiten. Sie werden durch *die jüngste (insubrische) Aufwölbung* zu der Kette der heutigen Alpen emporgehoben. Dadurch dass in diesem reifen Entwicklungsstadium keine weitere nennenswerte Wanderung der Vortiefe nach Norden mehr stattfindet, überholt der Undationsimpuls das Deckenpaket. Diese Welle läuft sich tot, wenn man davon ausgeht, dass der Zyklus von Mesoundationen an eine bestimmte Zone, das primäre geosynklinale Becken, gebunden ist.

Die Südenden der Decken, die anfänglich auf dem Nordflügel oder dem Gipfel der Undation lagen, kommen jetzt auf dem Südflügel zu liegen und erhalten im Zusammenhang damit ein entgegengesetztes Gefälle (in der Richtung der lombardisch-piémontischen Innensenke). Dies verursacht ein Abgleiten in umgekehrter Richtung, die sog. *„Rückfaltung"*. Die Rückfaltung gibt den alpinen *„Wurzeln"* ihr Antlitz. *Diese stellen jedoch nicht die Narbe der ausgequetschten Decken dar, sie sind vielmehr die ausgezogenen Teile des ursprünglich emporgehobenen Hinterlandes des alpinen Deckensystems, die erst später, nach Ablauf der Deckenbildung, steilgestellt und zurückgestülpt wurden.* Dass die Gesteinspakete in der Wurzelzone dünner

sind als im Frontalteil ist dann auch nicht die Folge einer weitgehenderen Zusammenpressung; die Pakete erfuhren hier im Gegenteil während des Abgleitens nach Norden im Anfang eine Zerrung oder flossen auseinander, während sie an der Front zusammengeschoben und verdickt wurden. Die im Vergleich mit der Stirn der Decken in der Wurzelzone intensivere Metamorphose hängt mit dem Aufdringen der postalpinen Batholiten in der Zerrungszone des Wellengipfels zusammen.

Unsere Kenntnis von der *Geologie der Südalpen*, südlich der sog. Wurzelzone und westlich von Triest, ist in jüngster Zeit durch die Tätigkeit der ESCHER'schen Schule in Leiden wesentlich erweitert worden. Die Auffassung STAUB's wonach hier die südwärts gerichteten Schuppen auf Rückfaltung zurückzuführen sind, hat sich im wesentlichen bestätigt. Die permischen Porphyre, sowie die tertiären Intrusiva und Extrusiva (Adamello, Predazzo, vizentinische Berge, Euganeen, steirischen Andesite) verleihen diesem Teil der Dinariden den Charakter der vulkanischen Innenzone eines vorherrschend nordwärts zusammengeschobenen Gebietes. Man darf diese Ast sicherlich nicht mit dem im weiteren Verlauf der Dinariden von Triest südostwärts streichenden Bögen von Dalmatien und Albanien gleichstellen. *Die vulkanische Innenzone der Alpen hat ihre natürliche Fortsetzung in der vulkanischen Innenzone der Karpathen.* Wie schon ESCHER (1911) und JENNY (1924) betonten, beginnen die Dinariden in Wirklichkeit erst bei Triest.

Aus den ausgezeichneten Studien von WINKLER (1926, 1928) ergibt sich, dass *südlich von Triest im Tertiär in das vom alpinen Undationszyklus durchquerte Gebiet von Osten her eine undatorische Welle eingedrungen sein muss, die aus dem südwestlichen Teil der pannonischen Innensenke stammt.* Die gewaltigen Knickungsüberschiebungen in den östlichen Südalpen sind nach WINKLER vermutlich nur die Folgen der *Einfügung des kretazischen Alpenbogens in den jüngeren, tertiären Bauplan der Dinariden.*

In gleicher Weise drang von Südwesten her (aus dem tyrrhenischen Zentrum), die nordapenninische Undation in die Innensenke der alpinen Wogen ein. Die Verfrachtung der ligurischen Decke ist posteocän (WIJKERSLOOTH 1930, TEICHMÜLLER 1932). Sie hat also erst stattgefunden, nachdem die Gosauaufwölbung bereits wieder niedergebrochen und nordwärts gewandert war. Die Flyschvortiefe der Apenninen war zugleich die Rücktiefe der alpinen Flyschaufwölbung, die ungefähr im Pogebiet gelegen haben muss. Wahrscheinlich waren darum dieser Vortiefe besonders schroffe Höhenunterschiede eigen, wodurch die 200 km weite Überschiebung der ligurischen Decke zustande kommen konnte, die nur eine Dicke von höchstens 2 km besitzt. Möglicherweise liegt hier eine Reliefüberschiebung vor, wobei die ligurische Decke wie ein Gletscher weithin über den damaligen Meeresboden abgeschoben ist.

Mit dem Schneiden des Westalpenbogens mit der nordapenninischen Vortiefe hängt die Bildung einer grossartigen *Rückfaltung des südlichsten*

Teils des Westalpenbogens in der Umgebung von Genua zusammen (die sog. *„Narbe von Genua"*).

Die Adria ist nach dieser Auffassung nicht der versenkte Sporn des afrikanischen Kontinents. Sie stellt vielmehr die ursprüngliche Innensenke des Alpenbogens dar, worin dann von Osten und Westen die Dinariden und die Apenninen eingedrungen sind, wodurch sie wieder zur Vortiefe neuer Undationen geworden ist. So dürfte vielleicht das Raumproblem des Alpenbogens eine ungezwungene Erklärung finden.

In den ausserhalb der alpinen Geosynklinale i.e.S. gelegenen Sedimentationströgen kann bereits während des Einsinkens stellenweise Volltroggleitung stattgefunden haben; die Hauptbewegungsphase fällt jedoch erst in die Zeit, in der die genannte Senkungszone emporgedrückt wurde und ihr Inhalt sich über das gleichzeitig einsinkende Vorland ergoss. Die Sedimentserien schoben dann, bisweilen mit „Keilen" des kristallinen Untergrundes, fächerförmig auseinander.

Auch zur Erklärung der Faltungs- und Überschiebungsstruktur dieser *Vorlandketten* ist es nicht nötig anzunehmen, dass die kontinentalen Blöcke den Inhalt der Sedimentationströge zusammen- und schliesslich auspressen. Ihre Tektonik ist mit Hilfe der bicausalen Auffassung primärer vertikaler Oszillationen und sekundärer Gleitbewegungen im Sinne HAARMANN's völlig zu begreifen.

Da die Sedimentationsbecken der Vorlandketten von kontinentalen Schollen eingeschlossen sind, kann sich der von ihnen ausgehende Undationsimpuls nicht zu einem seitwärts wandernden Undationszyklus entwickeln, wie die Impulse der zentralen Becken im geosynklinalen Gebiet i.e.S. Der ganze Zyklus beschränkt sich hier auf einen oder mehrere Hebungsimpulse des ursprünglichen Senkungsgebiets.

Wie in der Undationstheorie gezeigt wurde, ist die magmatische Differentiation sowohl unter der Geosynklinale als auch unter dem Kontinent aktiv, aber sie verursacht im Kontinentalgebiete nur unregelmässige Oszillationen[1]), und das Magma erreicht selten die Oberfläche. Als Beispiele für Gebiete mit *Vorlandvulkanismus* können das französische Zentralplateau, Süddeutschland und Marokko gelten.

In den Geosynklinalgebieten ist an der Innenseite des undatorischen Bogens die Kruste gedehnt bzw. gebrochen. Hier sind darum die Bedingungen für das Auftreten von äusserem Vulkanismus besonders günstig und bildet sich eine ausgesprochene *vulkanische Innenzone.*

Die atlantischen und pazifischen Magmatypen scheinen regellos über die vulkanischen Gebiete des Vorlandes und der Geosynklinale verbreitet zu sein.

Während der orogenetischen Revolution konsolidiert (STILLE) oder erstarrt (KOBER) die Geosynklinale zum Kontinent oder Kratogen. *Unter diesem Erstarren hat man nach der Undationstheorie die Abkühlung der*

[1]) "Germanotype" Orogenese im Sinne STILLE's (1924).

ursprünglichen Salsimaschmelze zu verstehen, wobei diese sich in Sal.und Sima trennte. Bei der Trennung wird latente chemische Energie frei und das gravitative Gleichgewicht in der Kruste gestört. Dies gibt Anlass zu Orogenese und Vulkanismus. Die gebildete *salische Kruste* ist an die herrschenden Bedingungen von T und P angepasst und besitzt daher keine latente chemische Energie mehr wie das Salsima: *Sie ist erstarrt und in diesem Sinne tot.* Die ursprüngliche Geosynklinale ist schliesslich als neue kratogene Zone mit dicker salischer Kruste den älteren kontinentalen Kernen einverleibt.

Übrigens ist die ganze Entwicklung der Erdkruste nach der Undationstheorie nichts anderes als *das Streben nach Gleichgewicht,* die *„Äquilibriopetalität".*

Bandoeng, im Mai 1933.

Anatomy. — *The Arterial Supply of the Cerebral Cortex in a Chinese Brain.* By A. RUBINO. (American Health Service, Hongkong.) (Communicated by Prof. C. U. ARIËNS KAPPERS).

(Communicated at the meeting of June 24, 1933).

The object of this investigation has been to ascertain the exact distribution of the branches of the major cerebral arteries in order to determine the relationship between the cortical areas of arterial supply and the areas of histological and macroscopic differentiation. The detailed description of the individual arteries has been given for the Chimpanzee by SHELLSHEAR (1930); and the branches of the anterior cerebral artery in man have been fully described by CRITCHLEY (1930). It has been decided to present the material in the form of an atlas with a brief description of each figure rather than to embark upon a tedious and long discourse. It is felt that these figures are sufficiently detailed and explanatory to fulfil their object of acting as a work of reference for clinicians and others working on the subject of the cerebral vasculature.

Plate 1. Figure 1. The line of demarcation between the anterior and middle cerebral arteries is shown. It reaches the dorso-lateral surface anteriorly at the outer end of the sulcus fronto-marginalis of WERNICKE so that the whole of this sulcus lies within the anterior cerebral area. The sulcus fronto-marginalis of EBERSTALLER lies within the middle cerebral area. The line now passes backwards to reach the sulcus praecentralis superior where this sulcus is joined by the sulcus frontalis superior. In this part of its course the line is seen to lie wholly lateral to the sulcus frontalis superior. The various branches of the anterior cerebral artery as they approach this line have a marked posterior direction and are each associated with definite elements of the sulcus frontalis superior. The triradiate sulcus, immediately anterior to the sulcus praecentralis medius, lies wholly within the middle cerebral area. The arterial relations of this sulcus and

the reasons for regarding it as the homologue of the sulcus arcuatus of the anthropoids have been fully discussed by SHELLSHEAR (1930).

The line of demarcation then crosses the sulcus centralis at approximately the boundary between the centres for leg and trunk movements and reaches the lower part of the sulcus postcentralis superior. The line then runs on the superior parietal lobule to end in the angle formed by the sulcus paroccipitalis and the inner arm of the sulcus parietalis transversus[1]). At this point on both sides there is a free anastomosis between the anterior, middle and posterior cerebral arteries. No anastomoses were found between contiguous branches of the major arteries but the figure shows the presence of moderately large anastomosing channels across the line of arterial demarcation between the branches of the anterior and middle cerebral arteries.

The line of demarcation between the middle and posterior cerebral arteries lies immediately medial to the sulcus paroccipitalis and crosses the medial part of the occipital operculum, under which the sulcus occipitalis transversus is hidden. Its relations to the sulci and to the area striata are better seen from the posterior aspect and will be dealt with when that is described. In the temporal region the line runs just lateral to the tentorial border and finally disappears on to the tentorial surface leaving the tip of the temporal lobe within the middle cerebral area. Along the tentorial margin there are anastomoses between the middle and posterior cerebral arteries.

Text figure 1 shows the chart of arterial areas constructed from the

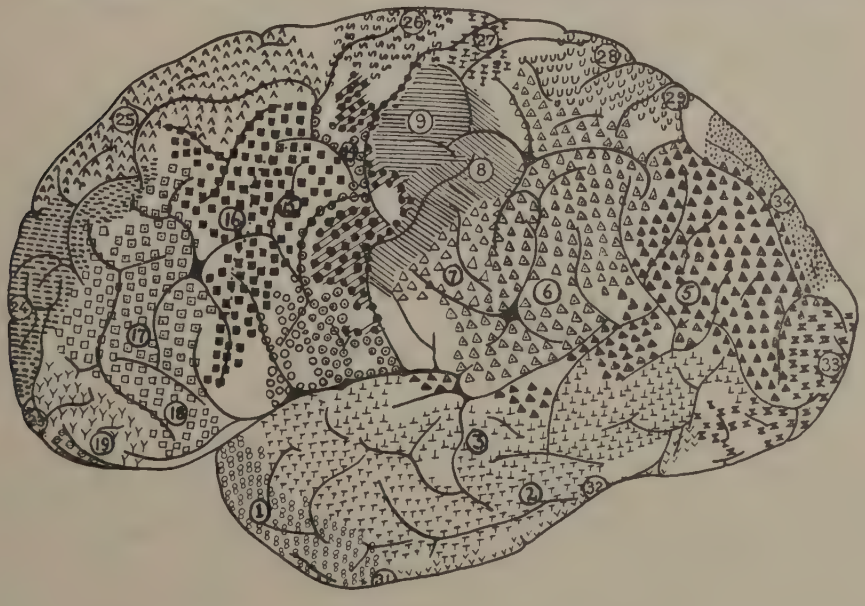


Fig. 1.

[1]) A term used by SHELLSHEAR to denote the anterior transverse element of the sulcus paroccipitalis.

arteries shown in Plate 1 fig. 1. These areas show a considerable measure of agreement with the areas in the Chimpanzee. The greatest differences are found in the greater expansion of the frontal and parietal regions. The distribution of the arteries to the central regions reveal a considerable degree of similarity. The only area which will be described in detail will be the area striata because the distribution of the line of GENNARI was ascertained.

Plate 1. Figure 2 shows the distribution on the medial surface of the left hemisphere. The arteries supplying the frontal region of the medial surface almost exactly agree with the description of the pattern in the

Fig. 2.

Chimpanzee and with CRITCHLEY's description in man. It is interesting in this brain to notice that the arteries on the medial surface of the frontal lobe do not pass into the depths of the sulci as they do in the parietal region and in general on the lateral surface. The precuneal region is supplied by branches of the anterior cerebral artery which anastomose freely with the parieto-occipital branches of the posterior cerebral artery. The medial surface of the occipital lobe will be dealt with when the posterior region is discussed.

Plate 2. Figure 1 shows the arteries on the dorsal surface. In this figure and in text figure 3 the lines of demarcation between the major arteries and the areas of vascular supply are shown. On the right side this line clearly points to the homologue of the sulcus rectus being the sulcus frontomarginalis of WERNICKE and the anterior part of the sulcus frontalis medius. On both sides it crosses the sulcus praecentralis close to its point of contact with the sulcus frontalis superior; and posteriorly it lies medial to the pars horizontalis and pars occipitalis of the sulcus intraparietalis.

A. RUBINO: The Arterial Supply of the Cerebral Cortex in a Chinese Brain.

Plate I

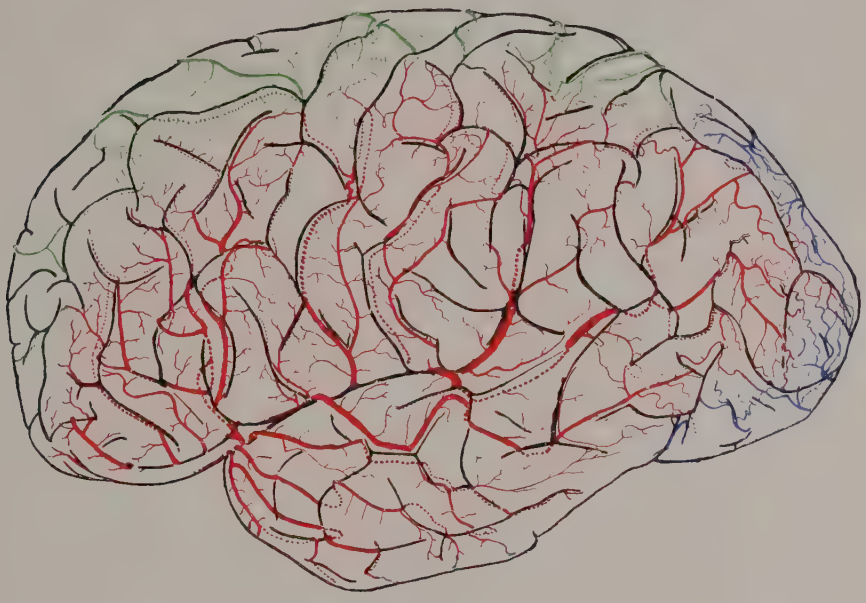


Fig. 1.

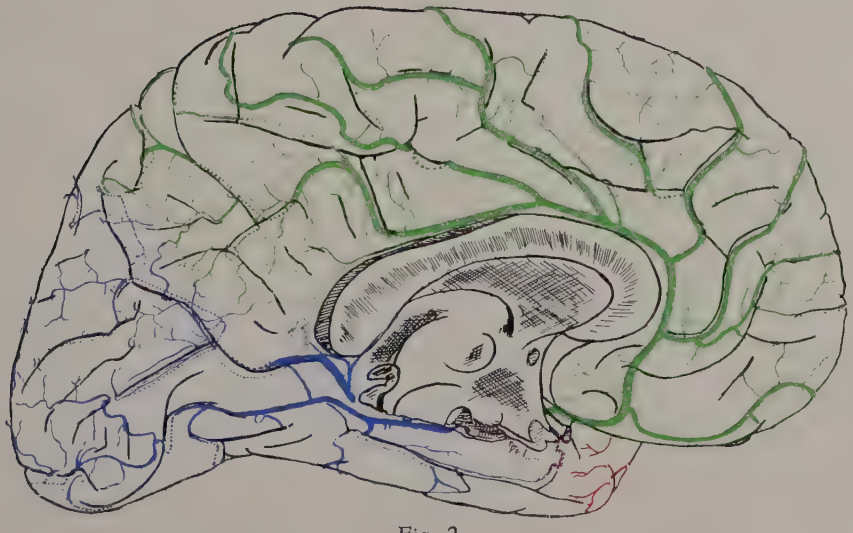


Fig. 2.

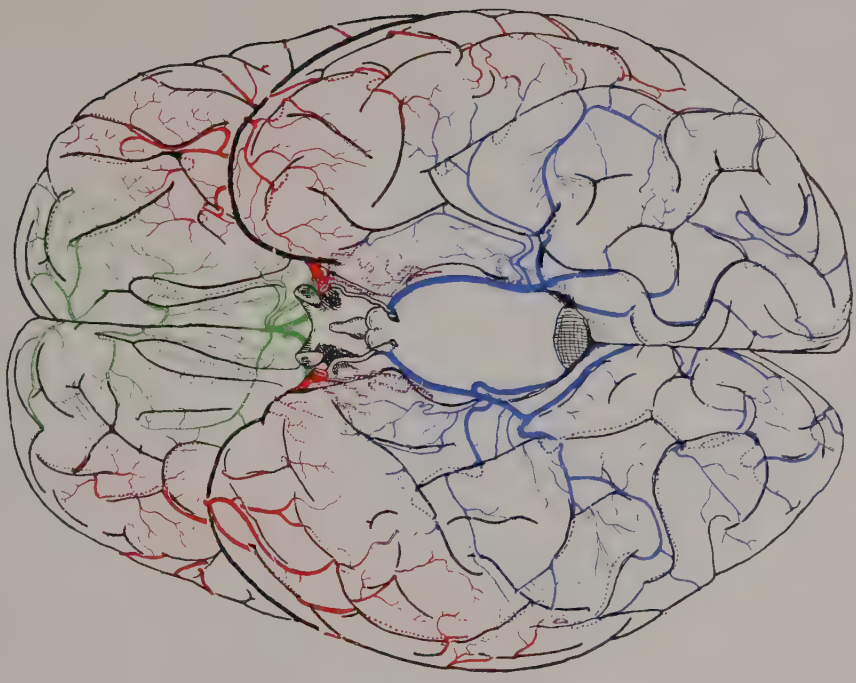


Fig. 2.

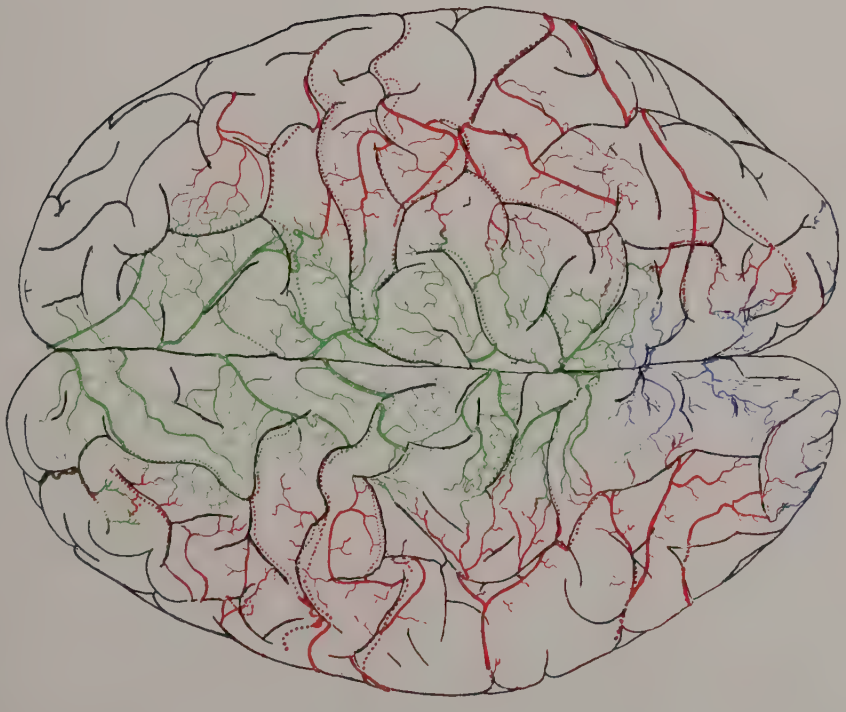


Fig. 1.

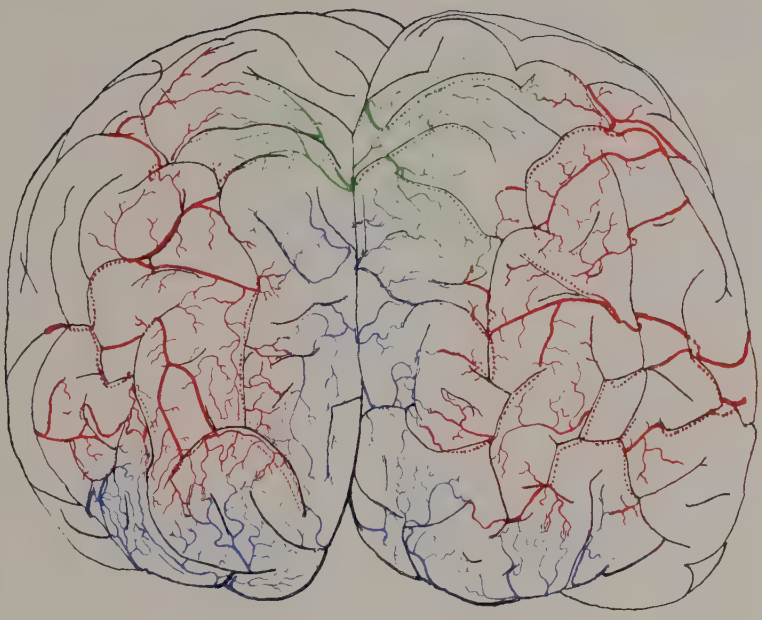


Fig. 1.

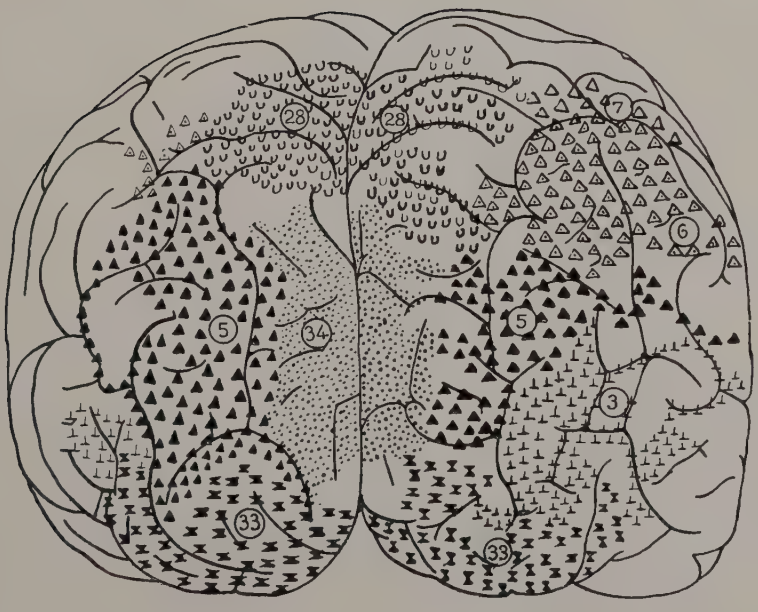


Fig. 2.

The point of junction of the three major arteries in the anterior part of the arcus parieto-occipitalis on both sides is an important landmark. The surface extent of the arcus parieto-occipitalis differs on the two sides as

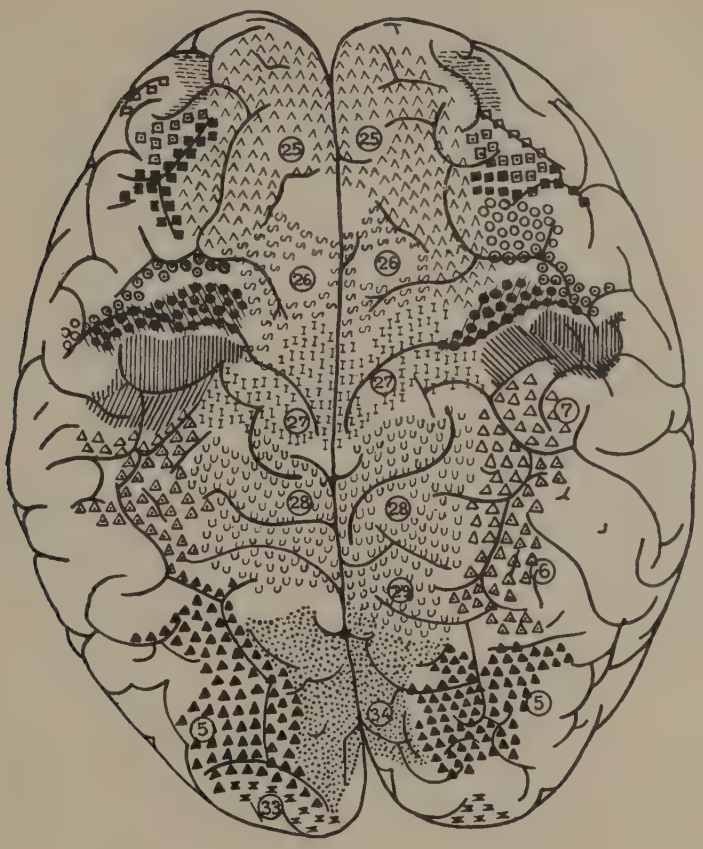


Fig. 3.

well as that of the lobulus postparietalis. These differences also find their expression in a corresponding difference in the arterial pattern.

Plate 2. Figure 2 shows the arteries of the inferior aspect of the brain. The corresponding vascular areas are shown in text figure 4. Whereas there is little indication in the literature beyond the papers by SHELLSHEAR on the brains of the Orang utan and Chimpanzee of the constancy of the line of demarcation on the lateral surface, there is shown in the works of DURET (1874), BEEVOR (1909) and in most text books of anatomy remarkable agreement in the position of the line separating the anterior and middle cerebral areas on the orbital surface of the frontal lobe. This brain shows that line in its constant position passing from the postero-internal arm of the sulcus orbitalis obliquely across to the antero-lateral arm and thence to the supraorbital border from which point we have traced it on the lateral surface. The position of this line on the orbital surface and its approximate

relations with the cortical areas of ELLIOT SMITH and BRODMANN have been discussed by SHELLSHEAR (1930) and are confirmed in this specimen. The boundary between the middle and posterior cerebral arteries on the tentorial

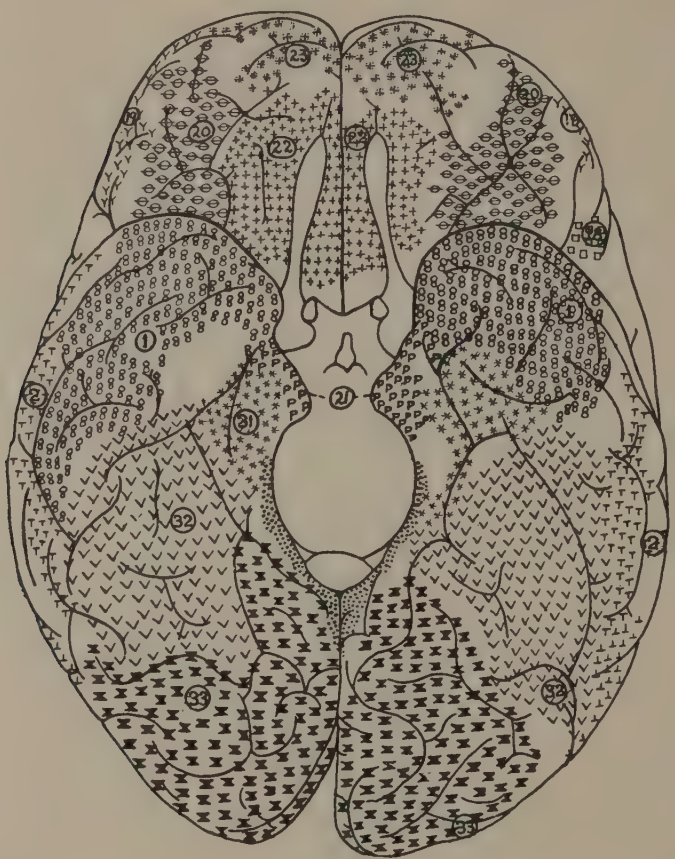


Fig. 4.

surface of the temporal lobe is likewise in agreement with the figures of DURET, BEEVOR and SHELLSHEAR. The position of this line in relation to the temporal pole confirms the area temporo-polaris of ELLIOT SMITH as an autonomous vascular area supplied by the temporo-polar branch of the middle cerebral artery. The uncus is supplied by both the anterior choroidal and posterior cerebral arteries.

Plate 3. Figure 1 and Plate 3 figure 2 show the arteries and vascular areas of the posterior aspect of the brain. Text figures 5, 6 and 7, show the sulcal pattern and the relations of the area striata. On the left side the sulcus occipitalis transversus is partly hidden under the lunate operculum. The sulcus lunatus shows a breaking into two parts by the gyrus translunatus as described by ELLIOT SMITH (1909). The sulcus praelunatus is continuous with the pars ventralis sulci lunati. The line of GENNARI falls short of both the pars dorsalis and the pars ventralis. The driving back of

the striate cortex from the lateral to the medial surface is in its earlier stages. On the right side the sulcus lunatus has broken down, the striate

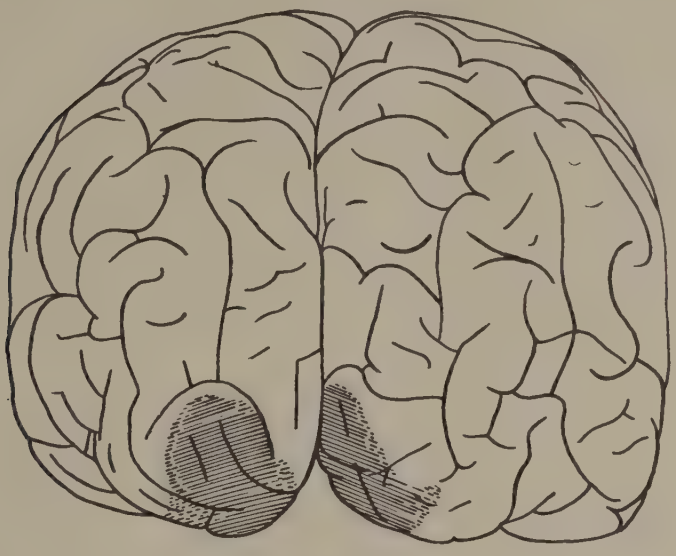


Fig. 5.

cortex approaches closer to the medial surface and is pushed somewhat vertically upwards producing a superior polar sulcus of BOLTON. The

Fig. 6. Fig. 7.

figures of plate 3 show that the line of vascular demarcation coincides with the anterior boundary of the area striata. The figures showing the

medial surface also show a close relationship between the vascular distribution and the extent of the area striata. Although the figures of the vascular areas in this region have been made to include under one conventional sign more than one area it was thought preferable not to attempt further subdivision but to leave them as a reference for further investigation. BEAUVIEUX (1929) has however divided up the branches to the occipital region and his classification is in accord with the distribution as shown in these figures. His paper was received since these drawings have been made. Particular reference is made to his paper because his figures show how the middle cerebral area encroaches more and more on the occipital region as the striate cortex retreats before it.

This work was carried out in the Department of Anatomy in the University of Hongkong. I have to acknowledge my indebtedness for the facilities which were afforded me there.

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Anatomy. — *The Arterial Supply of the Cerebral Cortex.* By JOSEPH L. SHELLSHEAR. (University of Hongkong.) — (Communicated by Prof. C. U. ARIËNS KAPPERS).

(Communicated at the meeting of June 24, 1933).

The constancy of the arterial supply to the structures of the hind- and forebrain has been established by many writers. The question of the variability of the channels of intermediate distribution need not be considered here because this has been fully discussed in my previous papers (SHELLSHEAR, pp. 1 to 6 (1929) and p. 187 (1927 a)).

STOPFORD (1915-16) established the constancy of the supply of many of the structures in the pons and medulla oblongata: "Invariably the trigonum hypoglossi was accurately defined by the stain of the injection of the anterior spinal artery" (p. 259); "The injection always demarcated the trigonum vagi - - - - when the vertebral artery was injected" (p. 265); and on the injection of the posterior spinal artery "the descending ramus sup-

plied the funiculi gracilis and cuneatus with their nuclei, and in all cases where the branch was present this distribution was found to be absolutely constant" (p. 273). Many of the findings of STOPFORD have been confirmed by me (SHELLSHEAR (1927 d and 1929)).

BEEVOR (1909), even by the method of differential injection found a considerable degree of constancy. He found that on injection the anterior nucleus of the thalamus stood out as an island, and that the level of the anterior commissure formed the meeting place of five separate arteries. In general his observations have been confirmed. I used his method to determine the arterial supply of the claustrum and, in so doing, was able to repeat many of his results (SHELLSHEAR 1920).

His opinion would seem to be that the arrangement of the arteries was without functional significance: "The fact that so small a body as the corpus subthalamicum can receive its blood supply from two different arteries is against the theory that the arrangement of the blood supply can have any functional significance, as it appears to be purely anatomical and not distributed according to the physiological action of the part" (p. 16). Even today the physiological action of the corpus subthalamicum is obscure. The functional significance of the arterial supply of any structure can only be measured in terms of the other morphological and physiological known facts.

As against BEEVOR's opinion, which is contrary to the views expressed by the writer in many papers, let us turn to the arterial supply of the lateral geniculate body concerning whose function we have considerable knowledge (BROUWER and ZEEMAN (1926), and others). ABBIE, in a paper at present in course of publication, shows that this body receives its arterial supply from both the anterior and posterior choroidal arteries and that this supply has great functional significance when compared with the morphological findings of BROUWER and ZEEMAN. He finds that the anterior choroidal artery supplies the lateral and anterior parts of the external geniculate body, that the posterior choroidal artery supplies the medial and posterior parts, and that the hilar region receives its supply from both arteries. The lateral and medial single supply is to the non-macular projection area and the double hilar supply corresponds with the macular projection area as defined by BROUWER and ZEEMAN.

Turning to the primitive olfactory areas, the pattern of the perforations in the region of the locus perforatus anticus made by the basal branches of the anterior and middle cerebral arteries is constant not only in the Primates but in the Vertebrata generally. The tuberculum olfactorium is supplied by both arteries. The basal branches of HEUBNER's artery, arising from the anterior cerebral artery, pass into the substance of the brain along the site of the erased endorhinal fissure to supply the head of the nucleus caudatus and other structures. The basal branches of the middle cerebral artery lie in series posterior to those of the anterior cerebral artery and supply palaeostriatal derivatives. Finally, more laterally placed basal

branches of the middle cerebral artery produce constant perforations and supply the claustrum, a hypopallial derivative (SHELLSHEAR 1920). It will be shown that this constancy has a definite functional significance ; and, further, that the principle of a functional supply is equally applicable to the arteries supplying the cerebral cortex.

In the crocodile (Fig. 1) the anterior spinal channel takes precedence over the vertebral channel in distributing blood to the higher parts of the brain ; nevertheless its branches are confined to the motor regions of the spinal cord and medulla oblongata, the main function of the channel itself being to enhance the distribution. The relations of the metamerically arranged branches of supply bear the same relations to the cranial nerves as they do in man. In the cerebrum the tuberculum olfactorium is supplied by both anterior and middle cerebral arteries, the pyriform area by the middle cerebral artery and the hippocampal areas by the posterior cerebral group of arteries. The newly developed primordium neopallii, interpolated between the primitive forebrain structures — pyriform and hippocampal — lies in the field of the junctional anastomoses between the major arteries. Just as the works of ELLIOT SMITH and others have revealed to us those areas of the human cerebrum which are primitive and those which are of later development, so also the study of the arteries in a phylogenetic series shows how the arterial supply to the new areas is derived from a more primitive pattern, the groundwork of which is still retained in man in relation to the primitive forebrain structures (SHELLSHEAR (1929), KAPPERS (1933)).

In the Echidna (fig. 2), with the increasing development of the neop-

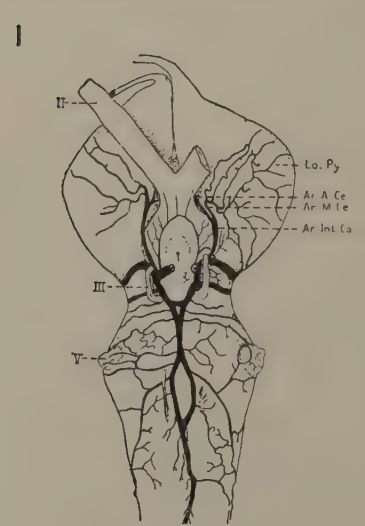


Fig. 1. The arteries on the ventral aspect of the brain in the Crocodile.

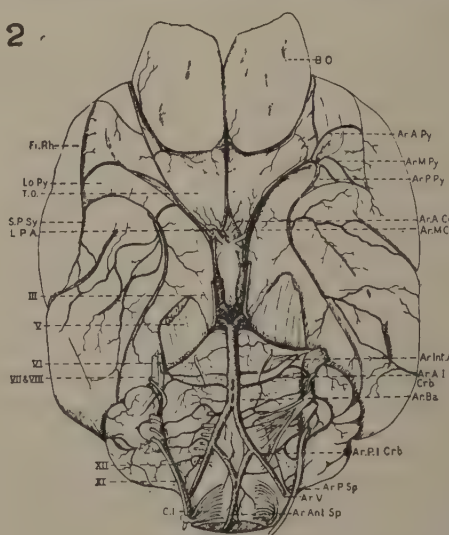


Fig. 2. The same in Echidna aculeata.

allium, the vertebral arterial channel enlarges to equal in size the anterior spinal arterial channel ; otherwise the supply to the hindbrain and the

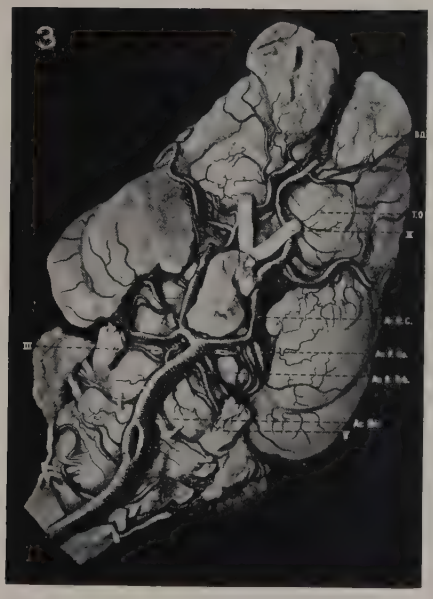


Fig. 3. The arteries on the ventral aspect of the brain in Myrmecophaga jubata.

relations of the branches to the nerves is the same. In the region of the tuberculum olfactorium the distribution is the same as in the crocodile and the supply to the developing neopallium is derived from the anterior, middle and posterior cerebral arteries and the anastomoses between them. The question of the preponderance of the posterior cerebral supply in the Echidna has been discussed elsewhere (SHELLSHEAR p. 32 (1929), KAPPERS (1933)).

In the lower primates the importance of the vertebral arterial channel, foreshadowed in the Echidna, becomes more fully appreciated. The anterior spinal channel is diminutive and plays little part in the distribution to the higher cerebral structures. The supply to the region of the tuberculum olfactorium is precisely the same as in the Echidna. The pyriform area is still supplied by the middle cerebral artery and the neopallium by the three major cerebral arteries and the anastomoses between them. In the primates, where the sense of vision plays a predominant part in cerebral functions the individual arteries to the different visual centres in the midbrain, thalamus and forebrain, originally arising as separate metamerically-arranged branches, become grouped together as branches from a common stem usually named the posterior cerebral artery (fig. 3). The details of these phylogenetic correspondences have been fully discussed by me in various papers ; and, therefore, consideration of the principles of arterial supply will be mainly confined to the description of the lines of arterial demarcation between the major arteries and to the supply of the visual cortex.

The Lines of Arterial Demarcation between the Major Cerebral Arteries.

It is not within the scope of this paper to describe in detail the individual branches of the cerebral arteries and their areas of supply. This has

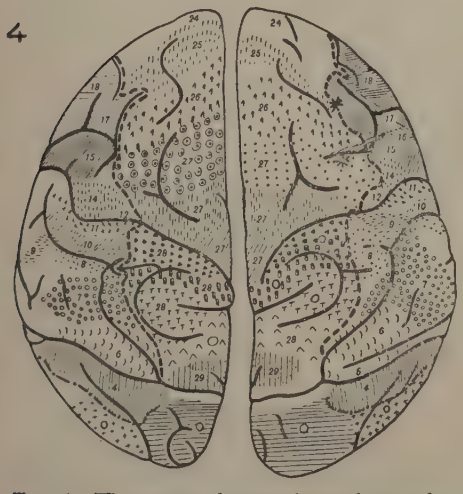


Fig. 4. The areas of arterial supply on the dorsal aspect of the brain in the Chimpanzee.

Fig. 5. The same on the ventral aspect.

already been done for the Orang Utan and the Chimpanzee by me (SHELLSHEAR, 1927 a, 1930). The figures published by RUBINO show an almost exact correspondence between the areas in the human brain and those described in the Orang Utan and Chimpanzee (figs. 4 and 5). Dr. ABBIE and the writer had the opportunity recently of examining the arteries in the brain of the Gorilla, thanks to the courtesy of Prof. ANTHONY of Paris. Figure 6 shows the areas of arterial supply for comparison with the other material already referred to. The detailed description of the individual areas is not necessary to establish the hypothesis that these areas are constant and have a functional import if it can be shown that the lines of vascular demarcation are constant between the major arteries; for if the major lines are not constant the minor areas related to those lines cannot be.

The line of demarcation between the anterior and middle cerebral arteries will be dealt with first. It extends from the interval between the double row of perforations at the anterior boundary of the locus perforatus anticus across the orbital surface of the frontal lobe and then runs about two centimetres from the dorsal border of the hemisphere on the lateral surface to end at the antero-lateral angle of the arcus parieto-occipitalis where the three major arteries meet. This line can be divided into three parts for descriptive purposes: 1. from the locus perforatus anticus to the supraorbital margin of the hemisphere, 2. from the supraorbital margin to the sulcus centralis and 3. from the sulcus centralis to the arcus parieto-occipitalis.

On the orbital surface the line passes obliquely forwards and slightly outwards to cut the supraorbital margin at a level corresponding with the junction of the horizontal and vertical portions of the sulcus fronto-marginalis. The inner two-thirds of the orbital surface lies within the anterior cerebral area and the outer one-third within the middle cerebral area (fig. 5). This line closely corresponds with the line of separation between the macroscopically determined areas orbitalis and praefrontalis posteriorly and the areas fronto-polaris and fronto-marginalis anteriorly (ELLIOT SMITH 1931). The significance of this line of demarcation on the orbital surface has been fully discussed by me in the papers referred to above. It is confirmed in the writings of DURET and recently by CRITCHLEY (1930).

The second part of the line has important morphological relations. After crossing the supraorbital margin it meets the outer end of the sulcus fronto-marginalis of WERNICKE where this sulcus is met by the anterior end of the sulcus frontalis medius. (The sulcus fronto-marginalis may be conveniently divided into three parts: internally, extending to the rostral keel, the sulcus fronto-marginalis of WERNICKE, externally, passing outward along the orbital margin, the sulcus fronto-marginalis of EBERSTALLER, and dorsally the anterior part of the sulcus frontalis medius constituting a vertical part of the threefold sulcus). The line of demarcation passes between the two fronto-marginal sulci where they are frequently confluent and then follows the sulcus frontalis medius for some distance.

In every brain in which the anterior part of the sulcus frontalis medius is well defined it forms the line of demarcation between the anterior and middle cerebral arteries. The detailed study of the arterial relations in conjunction with a study of the macroscopic and microscopic areas as determined by ELLIOT SMITH and BRODMANN reveals the homology of the sulcus rectus in man as the sulcus fronto-marginalis of WERNICKE and a part of the sulcus frontalis medius. In the general expansion of the frontal lobe the sulcus rectus is pushed forwards to the frontal pole and forms anteriorly an axial sulcus within the frontal keel (SHELLSHEAR 1930).

Leaving the sulcus frontalis medius the line of demarcation passes backwards to reach the sulcus centralis at the lower limit of the excitable area of the motor cortex for leg movements. The sulcus frontalis superior in its posterior part forms with the sulcus praecentralis superior a triradiate sulcus in the apes and this pattern is frequently retained in man. The line of demarcation passes through the triradiate sulcus at its central point. As a general rule the sulcus frontalis superior lies within the area of the anterior cerebral artery. Where this is not the case it is due not to vascular inconstancy but to irregularity in the arrangement of the sulcus frontalis superior (see SERGI 1913).

The morphological features which the constancy of this part of the line would appear to establish may be summed up as follows:

1. The new areas of expansion in the frontal lobe appear to be mainly confined to the area of the middle cerebral artery. Areas 44, 45 and 46 of BRODMANN — areas frontalis inferior and frontalis B of ELLIOT SMITH — derive their arterial supply from the external frontal branches of the middle cerebral artery. These vascular areas are larger in the human brain than they are in the anthropoids and point to the main expansion taking place in front of the centres for head and arm movements (SHELLSHEAR p. 83. 1930).

2. Areas 4, 6, 8, and 9 of BRODMANN are vascularised by both the anterior and middle cerebral arteries. The anterior cerebral supply is confined to the centres apparently concerned with leg movements. The centres for movements of other parts of the body are vascularised by the middle cerebral artery.

3. Comparison of the cortical charts of BRODMANN with the charts of vascular areas shows that the expansion of the lateral frontal region has caused the areas 4, 6, 8 and 9 to be thrust upwards and backwards. In consequence of this the general direction taken by the arteries is more posterior in man than in the anthropoids. This is well illustrated in the branches of the anterior cerebral artery.

4. The supply of areas 6, 8 and 9 of BRODMANN by the internal and external frontal branches of the anterior and middle cerebral arteries respectively shows a very close parallelism between the areas of arterial supply and the areas of histological differentiation (SHELLSHEAR 1930, CRITCHLEY 1930).

5. Anastomotic channels are found across the line of demarcation where the areas of BRODMANN cross the same line, but no anastomoses take place between contiguous branches of the major cerebral arteries. This confirms BEEVOR's observations and accounts for the apparent contradiction that he found the areas of the major cerebral arteries variable whereas I have found them constant (BEEVOR 1909). The hypothesis is put forward that anastomoses tend to occur between arteries of equal functional value.

The posterior part of the line of demarcation extends from the sulcus centralis to the antero-lateral angle of the arcus parieto-occipitalis where the fields of the three major arteries meet and where the arteries anastomose with one another.

The line lies between the sulcus postcentralis superior and the sulcus postcentralis inferior when these two sulci are found separate; and at their approximate point of junction when they are confluent. Posterior to this the line lies wholly within the superior parietal lobule. In every case the sulcus intraparietalis, excepting the sulcus postcentralis superior, lies within the field of the middle cerebral artery. The branches of the middle cerebral artery pass into the depths of the sulcus intraparietalis to emerge on the surface of the superior parietal lobule where they anastomose with branches of the anterior cerebral artery.

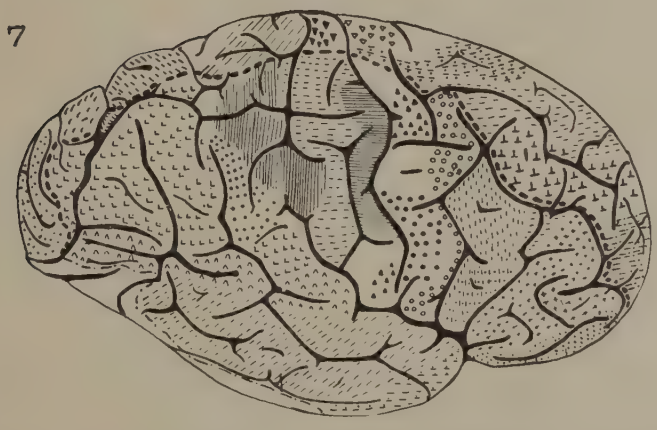
The line of demarcation is closely related with ELLIOT SMITH's visuo-tactile band passing between the peristriate and postcentral areas. In connection with this band it is to be noted that the expansion of the inferior parietal lobule which takes place below it to produce areas 39 and 40 of BRODMANN takes place entirely within the area of the middle cerebral artery.

The Line of Demarcation between the Middle and Posterior Cerebral Arteries.

The posterior cerebral artery of human anatomy is the artery of vision par excellence. The line of demarcation separates the visual cortex from the cortex lying anterior to it. The line of demarcation defines the area striata anteriorly as an autonomous vascular area, to use CHARCOT's term as evidence of his acceptance of DURET's statement that the fields of supply of the cerebral arteries are constant (1874). Starting from the arcus parieto-occipitalis the line passes backwards within the arcus and medial to the sulcus paroccipitalis as far as the inner part of the pars dorsalis sulci lunati. (This sulcus is well defined in RUBINO's figures which are made the basis for the description which follows.) The line now passes downwards and forwards within the curve of the sulcus lunatus to cross the pars ventralis sulci lunati near its point of continuation into the sulcus praelunatus, after which it runs close to the tentorial border almost to the temporal pole when it turns sharply on to the tentorial surface. The

relations of the line to the temporal lobe have been fully treated by me in the papers quoted.

To return to the occipital region of the line, BEAUVIEUX and RISTICH-GOELMINO (1926) show that the line of demarcation is much closer to



Figs. 6 and 7. The areas of arterial supply on the lateral aspect of the brain of a Gorilla and of an Australian aboriginal respectively.

the occipital pole than I have described above. A comparison of their pictures with RUBINO's in the brain of the Chinese, with the Chinese described by me (1929b), with the Orang utan (1929a), with the Chimpanzee (1930) and with the Australian brain here depicted (fig. 7) shows that not only is the method of supply constant but also that the intrusion of the middle cerebral supply into the occipital region is an exact parallel of the morphological changes which have taken place in the retreat of the area striata towards the medial surface. Where the occipital or lunate operculum is intact as in the Australian brain depicted the arterial pattern agrees with the cortical pattern; when the lunate sulcus breaks into its pars dorsalis and pars ventralis the gyrus translunatus separating them becomes vascularised by the middle cerebral artery; and finally when the sulcus

lunatus becomes obscured by the retreat of the striate cortex the area of the middle cerebral artery follows that retreat. These features are well shown in RUBINO's figures. It is clear from this that the distribution of the arteries must always be examined in conjunction with the morphology of the part supplied and that an average or schematic rendering of the pattern cannot give a correct idea of the functional distribution to serve as a basis for clinical interpretation.

There is a free anastomosis between the middle and posterior cerebral arteries across the line of demarcation but I have never seen any anastomosis between contiguous vessels. It is necessary however to point out that BEAUVIEUX and RISTICH-GOELMINO depict in their figure 1. such an anastomosis. The free anastomosis between the middle and posterior cerebral arteries may furnish the explanation of the escape of macular vision in cases of injury to the striate cortex. It is however to be noted that the area striata in the region of the sulcus retrocalcarinus often receives a blood supply distinct from the remainder of the striate cortex and in these cases the arterial pattern is very similar to the cortical localisation of BROUWER (1930).

The Line of Demarcation between the Anterior and Posterior Cerebral Arteries.

This line is found in the posterior part of the precuneal region and actually includes the extreme posterior end of the corpus callosum in many cases. Whether this is always the case I have not seen sufficient material to say. LEY (1932) shows the extent of the supply in his figure 2. In this region there is always a free anastomosis between the anterior and posterior cerebral arteries. It is referred to here, not only as a subject for further investigation, but also on account of its important bearing on the question of the existence of crossed connections between the lateral geniculate body and the striate cortex (POLIAK 1932).

The Principles of Arterial Supply.

In many papers I have discussed the areas of supply of the individual cortical arteries and so have thought it unnecessary to deal with these areas of autonomous arterial supply in a restatement of the principles previously put forward. These principles are stated as hypotheses to serve as a basis for further work since they cannot be more definitely established until further advances in morphology and physiology are carried out. The interpretation of the arterial distribution cannot precede the knowledge of the morphological facts although it may throw confirmatory light on them or may even suggest other avenues of investigation. The following hypotheses are put forward:

1. The arteries of the body are to a limited extent variable in their origin and intermediate course but are constant in their final areas of supply (DURET 1874, CHARCOT 1883).

2. The final areas of supply are constant because the supply has a definite functional significance (HILTON 1880).

3. Nerves, although variable in their intermediate course, are constant in their final terminations (JOHN HUNTER 1837). This is equally applicable to the somatic and sympathetic systems. It necessarily implies that the organ supplied has the same degree of constancy as the nerve supplying it. In other words nerve-organ units are constant in phylogeny and ontogeny. (SHELLSHEAR p. 5, 1929).

4. Nerve-organ units are subordinate parts of higher functioning mechanisms. The constancy of arterial supply must therefore presuppose a functional relationship between somatic and sympathetic nerve-organ units.

5. The problem of the development of nerve and muscle from a common neuromuscular mechanism is mentioned here without controversial intention (DART and SHELLSHEAR 1922).

The distribution of the arteries to the cerebral cortex is in harmony with these hypotheses. KAPPERS (1933), though calling attention to some variations that may exist in the vascularisation of the cortical facialis centre in comparing the lowest mammals with the primates, has confirmed the detailed description of the supply in the ECHIDNA and CRITCHLEY (1930) has confirmed in general my description of the anterior cerebral artery in the Chimpanzee. Nevertheless CRITCHLEY says: "Although there is an approximate correspondence between the territories of cerebral vasculature and the topography of various cytoarchitectonic zones, the idea of the functional significance must not be strained. Such schematization would presuppose a simplification of the anatomical basis of cerebral function which is out of accord with our conceptions of neurophysiology." I would reply:

1. That the hypothesis of a functional significance is justified from the exactitude of the arterial supply in so many parts of the body. 2. That a hypothesis only becomes strained when it is insisted upon as a law. Such an example is to be found in the unproved law that the whole nervous system is ectodermal in origin. 3. That the interpretation of the arterial distribution has not gone beyond the knowledge already acquired in other fields. 4. That an intimate and accurate knowledge of a constant arterial supply does not necessarily simplify the major problems. 5. That the areas of arterial supply depicted are in no sense schematic. 6. That it is possible that, when we have more detailed knowledge of the relation of the cortical fields to the sulcal pattern, the approximate correspondence between arterial fields and cortical fields will become a more exact correspondence.

In conclusion I have to express my thanks to Prof. ARIËNS KAPPERS for the opportunity of working in his laboratory and for advice and assistance which he has given me.

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Psychology.—*Mengentäuschungen im taktil-kinaesthetischen Gebiet* [1]). Von M. P. DE BRUYN OUBOTER. (Communicated by Prof. B. BROUWER).

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I.

Unsere räumlichen Wahrnehmungen vermögen in der Regel kein richtiges Bild über die räumliche Beschaffenheit der Dinge und über ihre Anordnung im Raume zu geben. Von dieser Erfahrung aus lässt sich jede Abweichung unserer Wahrnehmungen von den objektiven Verhältnissen, jede Diskrepanz zwischen Wahrnehmung und objektiv messbaren Raumgebilden als Täuschung bezeichnen. Die bekannten geometrisch-optischen Täuschungen

[1]) Arbeit aus dem psychologischen Laboratorium der Universität Amsterdam.

stellen bloss typische Beispiele solcher Täuschungen dar und weisen auf einige allgemeinere Tendenzen unserer raumwahrnehmenden und gestaltbildenden Funktion hin, die diese Abweichungen verursachen.

Bisher haben die Forscher ihr Augenmerk hauptsächlich auf die sog. geometrisch-optischen Täuschungen gelenkt. Seit den Untersuchungen von RÉVÉSZ weiss man indessen, dass den geometrisch-optischen Illusionen taktile, bzw. taktil-motorische Täuschungen entsprechen [1]). Wir dürfen daher die beiden Täuschungsgruppen mit dem gemeinsamen Namen „Raumtäuschungen" bezeichnen, womit wir zum Ausdruck bringen, dass für die Entstehung dieser Täuschungen dieselben Tendenzen verantwortlich zu machen sind.

Ausser den Raumtäuschungen gibt es in beiden Sinnesgebieten Lokalisationstäuschungen. Im Optischen liegt eine solche Täuschung vor, wenn beim Druck auf ein Auge ein Gesichtsobjekt sich scheinbar verdoppelt, indem es an zwei verscheidenen Stellen des Raumes wahrgenommen wird. Im taktilen Gebiet kennen wir zwei Arten von Lokalisationstäuschungen: einmal kann die berührte Hautstelle falsch lokalisiert werden, ein anderesmal wird dem Berührungsreiz eine falsche Raumstelle zugewiesen. Im ersten Falle können wir von Organtäuschungen (Hautlokalisation), im zweiten Falle von Projektionstäuschungen (Ortslokalisation) sprechen. Einen Übergang zwischen den beiden Arten stellen die bekannten Illusionen der Amputierten dar, wenn bei Reizung der Narbe am Amputationsstumpf eine Berührung an der fehlenden Extremität (dem sog. „Phantomglied") empfunden wird. Zu den Projektionstäuschungen gehört die bekannte Aristotelische Täuschung, die darin besteht, dass ein von zwei gekreuzten Fingern berührter Gegenstand so wahrgenommen wird als wären zwei räumlich getrennte Objekte vorhanden.

Die Lokalisationstäuschungen wurden bis jetzt nur bei taktiler Reizung, also bei ruhender Hand beobachtet, während bei bewegter Hand, beim Zusammenwirken des taktilen und kinaesthetischen Sinnes, solche noch nicht festgestellt wurden. Diese Lücke im System der Sinnestäuschungen wollen wir ergänzen, indem wir hier über einige Lokalisationstäuschungen berichten, die nur bei Bewegung der Tastorgane entstehen. Es handelt sich in unserem Falle um eine reine *haptische Projektionstäuschung,* bei der genau so wie bei der Aristotelischen Täuschung mehr Reizobjekte wahrgenommen werden als tatsächlich vorhanden sind.

II.

Grundversuch.

Es werden auf einer Holzplatte zwei erhabene, deutlich tastbare Punkte in einem Abstand von $2\frac{1}{2}$ cm montiert. Die Vp. hatte mit einem bzw. zwei Fingern darüber hin und her zu fahren.

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[1]) G. RÉVÉSZ, System der optischen und taktilen Raumtäuschungen. Kon. Akad. van Wetensch., Proceedings Vol. XXXII, N°. 8, 1929.

Wird ein Finger mit einer bestimmten mittleren Geschwindigkeit über zwei Punkte hin und her bewegt, dann nimmt man dem objektiven Tatbestand entsprechend zwei Punkte wahr. Führt man den Versuch mit zwei Fingern aus, so hängt alles davon ab, ob die Finger eng aneinander gepresst oder voneinander getrennt sind. Im ersten Falle funktionieren die beiden Finger wie ein einziges Tastorgan, folglich ändert sich an der Sachlage nichts. Werden dagegen die zwei Finger auseinandergespreizt und in der Weise bewegt, dass der eine Finger immer zwei, der andere nur einen Punkt berührt, so werden statt zweier Punkte deutlich *drei* beobachtet.

Bei genauerer Beobachtung zeigt sich, dass die drei Punkte räumlich ungleichmässig verteilt erscheinen: zwei liegen dicht nebeneinander, der dritte etwas entfernter. Manche Vpen fassten die beiden dicht nebeneinander stehenden Punkte als Verdoppelung oder Spaltung eines einzigen auf. Auch hatten die zwei ungleich alternierenden Abstände die Vpen gelegentlich zu einer rhythmischen Einstellung veranlasst.

Die Grenze, wo selbst bei getrennten Fingern noch zwei Punkte wahrgenommen werden, liegt für die meisten Vpen unterhalb 90 Doppelreibbewegungen in der Minute. Um die Grenzgeschwindigkeit herum werden indessen die Druckerlebnisse unbestimmter, man fühlt noch nicht ausgesprochen drei Punkte, sondern nur zwei, mit „etwas" dazwischen. Von da bis zu etwa 150 Doppelbewegungen in der Minute lassen sich deutlich drei Reizpunkte fühlen. Bewegt man die Finger schneller hin und her, dann werden nicht mehr drei, sondern mehrere Punkte, „eine kleine Menge" wahrgenommen, deren Anzahl aber nicht genau festzustellen ist[1]). Wird das Tempo noch mehr gesteigert, so geht die scheinbare Punktmenge in eine rauhe Fläche über.

Es hat sich also herausgestellt, dass bei Veränderung der Tempi folgende Stadien zu unterscheiden sind:

1. Bis zu einem bestimmten Tempo (etwa 90 Doppelreibbewegungen in der Minute) werden den objektiven Reizverhältnissen entsprechend zwei Punkte wahrgenommen,

2. bei einem schnelleren Tempo tritt die Mengentäuschung auf: man nimmt statt zweier drei Punkte wahr.

3. bei einem Tempo um 150 herum hat man den Eindruck einer zahlenmässig nicht schätzbaren Punktmenge; schliesslich erlebt man

4. bei einer noch grösseren Geschwindigkeit (um 200 herum) eine rauhe Fläche, in der keine diskrete Punkte mehr zu unterscheiden sind.

Mit der Veränderung der Tempi ändert sich nicht nur die wahrgenommene Anzahl der Punkte, sondern auch ihr subjektiver Abstand. Im allgemeinen besteht die Tendenz, bei schnellerem Tempo kleinere Abstände

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[1]) Die Schwierigkeit mehr als 3—4 disparate Punkte haptisch zahlenmässig zu erfassen, zeigt sich auch dann, wenn man mit einem Finger mehr als vier Punkte mit schnell bewegter Hand berührt.

zu empfinden. Die messende Bestimmung der scheinbaren Punktabstände ist mit grossen Schwierigkeiten verbunden. Bei langsamen Tempi wurde der Abstand von $2\frac{1}{2}$ cm bis auf 3.5 cm geschätzt, bei grösserer Geschwindigkeit verringerte er sich allmählich bis auf 1.2 cm. Der Punktabstand hängt auch von dem ausgeübten Druck ab, indem er bei stärkerem Druck grösser wird. Unter dieser Bedingung ist das dritte Stadium nicht mehr durch den oben beschriebenen Mengeneindruck charakterisiert, sondern man nimmt entweder den objektiven Verhältnissen entsprechend zwei Punkte wahr, oder eine Menge mit zwei deutlichen Akzenten.

Änderung des Abstandes der Reizpunkte und der Anzahl der Finger.

Eine Vergrösserung des ursprünglichen Abstandes von $2\frac{1}{2}$ cm auf 5 und 10 cm übt auf die Erscheinungen keinen wesentlichen Einfluss aus, die Beobachtungsbedingungen werden bloss ungünstiger. Bei einem Abstand von 5 cm und bei schnellerem Tempo hat man den Eindruck von zwei kleineren rauhen Flächen mit einem glatten Areal dazwischen. Beim Abstand von 10 cm werden unter denselben Bedingungen nur zwei Punkte empfunden. Verringert man den Abstand bis auf 1 cm, dann taucht das dritte Stadium schon bei einem langsameren Tempo auf.

Auch die Berührung der beiden Punkte mit drei bzw. mit vier Fingern übt auf die Täuschungserscheinungen keinen beachtenswerten Einfluss aus. Die Täuschungen werden nur weniger deutlich und prägnant. Je mehr Tastorgane in Bewegung gesetzt werden, um so geringer braucht die Geschwindigkeit der Fingerbewegung zu sein, um die Täuschung beobachten zu können.

Versuch mit drei Reizpunkten.

Berührt man statt zweier drei Punkte, dann tritt dieselbe Mengentäuschung auf, aber es werden statt dreier *vier* Punkte wahrgenommen. Die im Grundversuch festgestellten Stadien treten auch hier unter denselben Bedingungen auf. Im allgemeinen gilt der Satz, dass mit je mehr Tastorganen und Reizobjekten man operiert, um so langsamer das Tempo wird, bei dem die Täuschungserscheinungen schon auftreten. Wenn mit vier Fingern drei Punkte berührt werden, so treten die Täuschungen schon bei einer Geschwindigkeit von etwa 40 Doppelbewegungen in der Minute auf. Der Eindruck der rauhen Fläche wird schon bei ca 100 beobachtet.

III.

Wir haben festgestellt: wenn man zwei Finger in einem bestimmten Tempo derart über zwei Punkte bewegt, dass der eine Finger immer zwei Punkte berührt, während der andere nur einen, dann werden statt zweier Punkte deutlich drei wahrgenommen. Versucht man nun die wahrgenommenen Punkte auf die Finger zu lokalisieren, dann zeigt sich, dass man mit dem einen Finger zwei Punkte fühlt, mit dem anderen nur einen. Ferner lässt sich auch beobachten, dass, falls die Reibbewegun-

gen schneller werden, die Abstände zwischen den Punkten sich zu verringern scheinen[1]). Setzt man diese zwei Feststellungen miteinander in Beziehung, dann lässt sich die beobachtete Täuschung erklären.

Wenn ich in der oben beschriebenen Weise zwei Finger über zwei Punkte bewege, berühre ich tatsächlich mit dem einen Finger beide Punkte, mit dem anderen nur einen Punkt. Dass ich also drei voneinander getrennte Berührungen empfinde, ist kein Wunder. Dasselbe gilt auch für die Aristotelische Täuschung. Auch da berühren wir mit zwei Fingern zwei Punkte desselben Gegenstandes, folglich ist es selbstverständlich, dass wir zwei voneinander getrennte Berührungsemfindungen haben. Die Täuschung liegt also hier wie dort nicht darin, dass man getrennte Berührungen unterscheidet, sondern dass man *diesen Berührungsempfindungen eine entsprechende Anzahl von Reizobjekten zuordnet.* Wenn ich bei der Aristotelischen Täuschung die zwei disparaten Eindrücke auf denselben Gegenstand beziehen würde, dann bestünde keine Täuschung. Das gleiche läge vor, wenn wir in unserem Versuch den zweiten und dritten Punkteindruck mit dem objektiv zweiten Punkt identifizieren würden. Die Täuschung rührt also davon her, dass man *zwei* Tasteindrücke, die von *einem* einzigen Reiz ausgelöst werden, nicht identifiziert. Die Trennung entsteht infolge der Lokalisation des einen objektiven Punktes auf zwei verschiedenen räumlichen Stellen.

Es erhebt sich aber noch die weitere Frage: warum lokalisieren wir hier falsch? Die Erklärung ist einfach. Wie wir oben erwähnt haben, wird der subjektive Abstand zwischen den zwei Berührungspunkten an einem Finger bei einer schnelleren Bewegung geringer. Während diese zwei Punkte an dem einen Finger näher zu einander treten, kann sich die Lage des Berührungspunktes am anderen Finger nicht in demselben Mass ändern. Die Folge davon ist, dass der zweite Reizpunkt auf zwei verschiedenen Raumstellen lokalisiert wird. Diese doppelte Lokalisation eines objectiven Punktes verhindert die Identifikation der beiden Eindrücke in Bezug auf ein und dasselbe Reizobjekt, und dies ist der Grund der Täuschung.

Diese Abspaltung des zweiten Punktes wird noch deutlicher erlebt, wenn man denselben Versuch mit gekreuzten Fingern ausführt. In diesem Falle erscheinen nämlich die von beiden Fingern perzipierten Punkte in verschiedener Höhe übereinander, der räumlichen Lage der Tastfläche der beiden Fingern entsprechend. Wenn man den Versuch mit mehreren Punkten ausführt, wird die Erscheinung ganz besonders evident. Unter diesen Bedingungen wird nicht nur eine Identifikation unmöglich, sondern wegen des Höhenunterschiedes der Punktreihen verlieren die abgespaltenen Punkte gänzlich ihren subjektiven Charakter.

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[1]) Etwas ähnliches hat VIERORDT (Der Zeitsinn. Tübingen 1868, S. 118—123) beschrieben. Er konnte feststellen, dass wenn die Haut punktförmig gereizt und gleichzeitig ein Reizobjekt über die Haut hin und her bewegt wird, das Bewegungsareal bei steigender Geschwindigkeit zu schrumpfen scheint.